Dynamic Stochastic Model for Converging Inbound Air Traffic

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Weather accounts for the majority of congestion in the National Airspace System, which highlights the importance of addressing weather uncertainty to mitigate delays. First, this paper presents a new dynamic stochastic integer programming model that studies the single-airport ground-holding problem with respect to uncertainty in the separation between flights instead of airport acceptance rate. The current model is able to provide a more accurate schedule expressed in minutes for the individual flight. Second, a converging inbound air traffic model is formulated based on the current model. This paper addresses a problem involving the merging of two inbound streams into a single airport in which uncertainty in separation from minutes-in-trail restrictions is considered. Although the "first-come/first-served" policy is still obeyed by flights on the same path, the experimentation has shown that allowing flights on different paths to switch arrival orders can help reduce the total delays. Finally, in order to tackle the computational burden posed by the disaggregate integer model, this paper introduces a dual decomposition method to reduce computation time. The original problem is decomposed scenario by scenario into several subproblems; then, a parallel computing algorithm is developed to handle these subproblems. Such a combination increases the model's computational efficiency.

Nomenclature

$\mathcal{F}$	=	set of flight
f	=	index of flight
$\mathcal{H}$	=	set of required time separation profile scenarios
i	=	index of flight on path 1
j	=	index of flight on path 2
K	=	set of path
L	=	required flight time from arrival fix to airport, min
N_1	=	number of flights scheduled for path 1
N_2	=	number of flights scheduled for path 2
$P\{q\}$	=	probability of scenario q
q	=	index of scenario
Q	=	total number of scenarios
S_{ij}^q	=	reversal decision variables for flight i on path 1 and flight j on path 2 under scenario q
T	=	planning time horizon of the problem, min
$T_{k,f}$	=	time window for each flight, min
$\underline{T_{k,f}}$	=	first time period in the set $T_{k,f}$
t	=	time step
Y_q	=	array to express all Y decision variables under scenario q
$Y_{k,f,t}^{\text{plan}}$	=	scheduled arrival time at arrival fix
Γ	=	set of time periods
Δ	=	set of required time separation, min
$\Delta(t)^q$	=	specific time separation under scenario q, min
$\underline{\Delta}$	=	small separation, min
$\overline{\Delta}$	=	big separation, min
λ	=	cost ratio between one unit of airborne holding and ground holding
v_q	=	Lagrange multiplier for each scenario
$v_{k,f,t}^q$	=	Lagrange multiplier for flight f on path k at time t under scenario q
ξ_q	=	required time separation profile scenario q
τ_q	=	time where scenario tree diverges to produce a new branch

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I. Introduction

AIR traffic controllers use miles in trail (MIT) to allow aircraft to achieve on-time arrivals. MIT describes the minimum allowable miles between successive aircraft departing/arriving at an airport, over a fix, through a sector, or on a specific route. MIT is used to apportion traffic into a manageable flow, as well as to provide space for additional traffic (merging or departing) to enter the flow of traffic. For example, standard separation between aircraft in the en route environment is 5 n miles [1]. During a weather event, this separation may increase significantly. Many delays are directly attributable to MIT in an adverse weather event. A variation on MIT is minutes in trail (MINIT), which describes the minimum allowable minutes needed between successive aircraft. MINIT can be easily derived from MIT with a consideration of aircraft speed.

The ground delay program (GDP) is one of the most effective strategic traffic management initiatives used to alleviate congestion costs and it ensures safe and efficient air traffic [2–4]. A GDP is often issued to control air traffic volume to airports where the projected traffic demand is expected to exceed the airport's airport acceptance rate (AAR); the AAR describes the number of arrivals an airport is capable of accepting for a length/period of time (usually 15 min or more) [5,6]. A lengthy period of demand exceeding the AAR is normally a result of the AAR being reduced for some reason: most often, adverse weather. In a GDP, some flights are assigned a later time slot of arrival via ground delay to avoid airborne delay because it is cheaper and safer to delay flights on the ground than to hold them when they are airborne.

Weather accounts for the majority of congestion in the National Airspace System (NAS). Adverse weather such as fog, snow, wind, and reduced visibility may require greater separation between flights. Approximately 60% of the total delays in the NAS was caused by adverse weather in 2009 [7]. The imperfect weather forecast brings uncertainty into the air traffic management problem. Decisions made under uncertainty can cause airborne delays when the minimum separation[§] between flights is greater than the original forecast. If the forecast is too conservative, unnecessary ground delays will happen. This highlights the importance of addressing weather uncertainty to mitigate delays.

In the past two decades, the ground-holding problem (GHP) has been studied by many researchers to support GDP action at airports. The objective of this problem is to minimize the sum of airborne and ground delay costs. Most GHPs focus on modeling a response to a reduced AAR (landing capacity) caused by adverse weather. Efforts

[§]The required separation between flights in this paper always mean the minimum required separation.

to tackle GHP problems date back to 1987, when Odoni was among the first to systematically describe this [8]. Hoffman and Ball proposed a deterministic model for the single-airport ground-holding problem (SAGHP) with banking constraints, which added the constraint that flights must arrive within prespecified time windows. Such a condition is useful to model hub–spoke operations at major airports [9,10]. For a stochastic model, Richetta and Odoni formulated a static stochastic integer programming (IP) model for the SAGHP [11]. Following this, Bal, et al. proposed a modified version of the static stochastic model for the SAGHP, which solves for an optimal number of planned arrivals of aircraft during different time intervals [12]. But, in both of them, ground-holding strategies were decided "once and for all" at the beginning of planning time horizon and the models could be rerun in a "rolling horizon" fashion to allow for some replanning [13]. Richetta and Odoni formulated a dynamic multistage stochastic IP model for the SAGHP to overcome this limitation [14]. In this dynamic model, the ground-holding decisions were made at the scheduled departure time of the flights instead of once for all at the beginning. However, the ground-holding decision still could not be revised after it had been made. Mukherjee and Hansen improved this dynamic model by allowing for ground-holding revisions contingent on scenario realizations [15]. In all of the aforementioned models, the uncertainty in landing capacity was represented through a finite number of scenarios arranged in a probabilistic decision tree. As time progressed, branches of the tree were realized, resulting in better information about future capacities [16].

On the other hand, a 0–1 IP model was proposed by Bertsimas and Stock-Patterson, known as the Bersimas/Stock-Paterson model [17,18]. This model was formulated to address the air traffic flow management problem, but it can also handle the GHP as a special case. This model is a Lagrangian model, which is based on the trajectories of each individual aircraft. A limitation of Lagrangian models is that the dimension of this model is related to the number of aircraft involved in the planning time horizon. Bertsimas and Stock-Patterson proved that the 0–1 IP problem was nondeterministic polynomial-time (NP) hard by deriving the equivalent job-shop scheduling problem. Gupta and Bertsimas improved this model to address the capacity uncertainty [7]. This approach is addressing the uncertainty from the robust optimization aspect and solves the "worst case" in the same fashion as the deterministic one.

In summary, almost all existing GHP models in the literature are formulated accounting for the landing capacity (AAR) constraints, which encounter a few limitations in general, including the following:

1) It is difficult to schedule an individual flight accurately due to the relatively large time interval (normally 15 min or more).

2) It is difficult to handle arrival merging flows with changeable arrival order.

To overcome these limitations, this paper will consider the minimum safe separation requirements instead of landing capacity constraints. In fact, the separation requirement, which could be modeled using the separation criticality index [19], is a key factor for the controller workload, which is the main factor limiting the landing capacity [19]. The appropriate separation could be chosen by air traffic controller to deliver a prescribed number of aircraft per unit time to the airport runway to meet the AAR. Although a distribution over separation times in the form of a scenario tree is also needed, our model can provide a more accurate schedule expressed in minutes for the individual flight and could handle arrival merging flows with changeable arrival order.

In this paper, a dynamic stochastic optimization model is formulated by using linear stochastic programming, which can use dynamic updates of information about the minutes-in-trail separation in a single airport. This paper models the GHP with respect to uncertainty in the minimum separation between flights instead of AARs or landing capacity. Uncertainty in separation according to different weather conditions is represented through a scenario tree. This model is able to handle the time-varying required separation and the uncertainty rising from the imperfect forecast of weather conditions. We also address a problem involving the merging of two inbound streams into a single airport in which uncertainty in separation from MIT restriction is considered. Allowing flights on different paths to switch arrival order will help reduce total delays. Finally, we present a decomposition method for the stochastic problem modeled by the scenario tree method, in which the stochastic problem can be decomposed scenario by scenario to improve the computational efficiency.

The rest of this paper is organized as follows. Section II introduces the formulation of a stochastic dynamic model for converging two paths' inbound flights into a single airport. The model can handle the uncertainty in the minimum required separations and is adaptive to updated information as time progresses. After the model, a small size problem is used to demonstrate how our model works. Section III describes the dual decomposition method used to solve the large-scale stochastic optimization problem based on the scenario tree method. In Sec. IV, the numerical application results are presented and a discussion of the results follows. Finally, we summarize the conclusions in Sec. V.

II. Dynamic Stochastic Model for Converging Inbound Air Traffic

In this section, we present the development of the dynamic stochastic model for converging inbound air traffic. We consider two sets of flights that are scheduled to fly to a single airport from two paths; each set of flights arrives at the airport via an arrival fix, where the arrival fix describes the first point of the arrival approach to the destination airport in the terminal area. Flights on same path obey the first–come/first-served policy, but they can change order with flights on the other path. Since the first–come/first-served policy paradigm may not allow slot substitutions, the airline business models are not fully incorporated in this model. For each flight, there is a time window (slot) for arriving at the arrival fix. Flights are planned to reach their arrival fixes at their schedule timed or later (but still in the time window). If the weather conditions are not good, the time separation between successive landing flights will be greater than normal and flights may face airborne holding at the arrival fix. Alternately, ground holding will be imposed to delay flights before their departure to avoid airborne holding, because airborne holding costs more than ground holding and it has higher safety risk.

A. Scenario Tree and Uncertainty Weather Model

Following Richeta and Odoni [14], Mukherjee and Hansen [15], Kall and Mayer [20], and Mukherjee [21], we use a scenario tree to represent the evolution of weather conditions at airports. A scenario tree is the most common method to handle uncertainty in the air traffic management problem [16]. The scenarios can be identified from historical data for separations under different weather conditions; then, these scenarios are combined into a probabilistic tree (scenario tree). Each node of the tree represents a status of weather. As time progresses, each branch of the tree becomes a distinct scenario. For a detailed discussion of scenario generation and scenario dispersion, the reader is referred to the work of Liu et al. [16]. Let $\mathcal{H}$ denote the set of required time separation profile scenarios, and a scenario $\xi_q \in \mathcal{H}$ will occur with a probability $P\{q\}$. We assume that, in the beginning of the time horizon ($t = 0$), there are Q alternative scenarios, with each scenario providing a possible time-varying required time separation profile forecast for the entire time interval [0, T]. So, each node of the tree represents the time separation at that time. Let τ_q denote the time where the scenario tree diverges to produce a new branch. Figure 1 shows our notation using the scenario tree representation.

We assume that weather only affects the required time separation in the airport, i.e., adverse weather will increase the required time separation. Let Δ denote the set of required time separations. The required time separation is time varying and different from scenario to scenario (thus, our model is dynamic and stochastic). We use $\Delta(t)^q$ to denote the specific time separation. For simplicity's sake, we assume that weather can only change once: from bad weather (big separation $\bar{\Delta}$) to good weather (small separation $\underline{\Delta}$). But, the exact timing of weather change is uncertain. For example, suppose we have

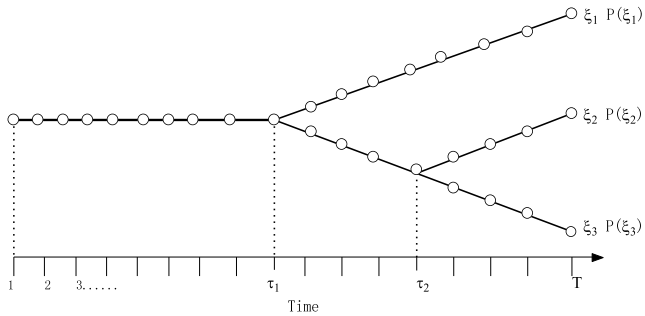

Fig. 1 Scenario tree representing the evolution of required time separation at the airport.

a time horizon of seven periods, i.e., $T = 7$. Moreover, there are three scenarios in the beginning: $Q = 3$. We assume the weather may change at $\tau_1 = 2$, $\tau_2 = 3$ and $\tau_3 = 4$. Let $\bar{\Delta} = 3$ and $\underline{\Delta} = 1$. Then, we have $\Delta^1 = (3, 3, 1, 1, 1, 1, 1)$, $\Delta^2 = (3, 3, 3, 1, 1, 1, 1)$ and $\Delta^3 = (3, 3, 3, 3, 1, 1, 1)$.

B. Problem Formulation

1. Notation

Let N_1 and N_2 be the number of flights scheduled for each path, and the path is denoted by the set $K = \{1, 2\}$. Let $T_{k,f}$ denote the time window for each flight. The time is a set of T time periods of equal duration, and it is denoted by the set $\Gamma = \{1, 2, \ldots, T\}$. Any flight from each path must pass the according arrival fix before landing at the airport, and the required flight time from the arrival fix to the airport is denoted by L. Here, we do not consider the difference of the required flight time from the different fixes to the airport; they are the same in this model. Let λ denote the cost ratio between one unit of airborne holding and ground holding. We consider $\lambda > 1$ because airborne holding is more expensive and we assume it is the same for all flights.

2. Decision Variables

The decision variables in the model are binary variables defined as follows:

$$W^q_{k,f,t} = \begin{cases} 1 & \text{if flight } f \text{ on path } k \text{ is landed by time } t \text{ under scenario } \xi_q \\ 0 & \text{otherwise} \end{cases}$$

$$Y^q_{k,f,t} = \begin{cases} 1 & \text{if flight } f \text{ on path } k \text{ arrives at arrival fix by time } t \text{ under scenario } \xi_q \\ 0 & \text{otherwise} \end{cases}$$

Note that our decision variables are similar to the Bertsimas/Stock-Patterson model; this definition using "by" instead of "at" is important to understand this model. Once flight f arrives at the fix or lands at the airport at time t, then variables will be set to 1 at time t and remains so for future time. We can record the status changing time as the arrival time or landing time. Here, Y variables represent quantities we can set in reality for this model.

3. Objective Function

The objective of the model is to minimize the expected combination cost of the airborne-holding delay and ground-holding delay for all flights:

$$\text{Min} \sum_{\xi_q \in \mathcal{H}} P\{q\} \Bigg\{ \sum_{k \in K} \sum_{f \in \mathcal{F}} \Bigg(\sum_{t \in T_{k,f}} (Y^{\text{plan}}_{k,f,t} - Y^q_{k,f,t}) \Bigg) + \lambda \Bigg(\sum_{k \in K} \sum_{f \in \mathcal{F}} \Bigg(\sum_{t \in T_{k,f}} (Y^q_{k,f,t} - W^q_{k,f,t+L}) \Bigg) \Bigg) \Bigg\} \tag{1}$$

where

$$Y^{\text{plan}}_{k,f,t} = \begin{cases} 1 & \text{if flight } f \text{ on path } k \text{ is planned to arrive at arrival fix by time } t \\ 0 & \text{otherwise} \end{cases}$$

Let $Y^{\text{plan}}_{k,f,t}$ denote the corresponding binary values of the scheduled arrival plan at the arrival fix, which is the prior information for this model and is deterministic and the same for all scenarios. In Eq. (1), the first component is the difference between the scheduled arrival time and actual arrival time at the arrival fix, which expresses the ground-holding delay. Note that, if a flight is reassigned to arrive later than its original scheduled time, we assume that the delay occurs at its original airport. We only consider the delay as a ground-holding delay and ignore the delay en route, since the airborne delay is more expensive than ground holding. The second component is the difference between the planned landing time and the actual landing time at the airport, which is the airborne-holding delay. The cost ratio λ is a multiplier that increases the penalty for airborne delay.

4. Constraints

Constraints (2) and (3) represent connectivity in time, which means that, if a flight has arrived (landed) by time t, then $Y^q_{k,f,t}(W^q_{k,f,t})$ will be set to one for all subsequent time periods:

$$W^q_{k,f,t} \le W^q_{k,f,t+1} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \tag{2}$$

$$Y^q_{k,f,t} \le Y^q_{k,f,t+1} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \tag{3}$$

$$W^q_{k,f+1,t+\Delta(t)^q} - W^q_{k,f,t} \le \sigma^q(t) \quad \forall k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \quad t \in \Gamma, \qquad t \le T - \text{in } f\Delta^q \tag{4}$$

where

$$\sigma^q(t) = \begin{cases} W^q_{k,f,t+(\bar{\Delta}-\underline{\Delta})} & t \in [\tau_q, \tau_q - (\bar{\Delta} - \underline{\Delta}) + 1] \\ W^q_{k,f,t-(\bar{\Delta}-\underline{\Delta})} & t \in [\tau_q + 1, \tau_q + (\bar{\Delta} - \underline{\Delta})] \\ 0 & \text{otherwise} \end{cases}$$

Constraints (4) represent the connectivity between flights in the same path. This constraint separates flights in the same path by required safety separation, depending on the weather condition. If one flight lands at the airport at time t, then the next flight from the same path must land after time$t + \Delta(t)^q$. Here, the required safety separation $\Delta(t)^q$ is time varying and different from scenario to scenario. The minimum separation is related to the weather conditions for that time period and would be chosen by the air traffic controller under experience.

The term $\sigma^q(t)$ on the right side works as a switch key at the weather changing time. It ensures that either the constraints before the weather changing time work or the ones after the weather changing time work. For example, we assume $T = 5$ and the weather may change at $\tau = 2$. Let $\bar{\Delta} = 3$ and $\underline{\Delta} = 1$. Then, we have $\Delta = (3, 3, 1, 1, 1)$; if we do not have $\sigma^q(t)$, our constraints look like following:

$$W^q_{k,f+1,4} - W^q_{k,f,1} \le 0 \tag{5}$$

$$W^q_{k,f+1,5} - W^q_{k,f,2} \le 0 \tag{6}$$

$$W^q_{k,f+1,4} - W^q_{k,f,3} \le 0 \tag{7}$$

$$W^q_{k,f+1,5} - W^q_{k,f,4} \le 0 \tag{8}$$

We can find that constraints (5) and (7) or (6) and (8) cannot be satisfied at the same time. Constraint (5) and (6) will make constraints (7) and (8) redundant because of constraint (2). In other words, if flight f does not land on time period 1, flight $f + 1$ cannot land on time period 4, even if the weather has become good and the separation is small on time period 4. So, if we want either of them to work, we just add the two pair constraints together, which makes it

$$W^q_{k,f+1,4} - W^q_{k,f,1} - W^q_{k,f,3} \le 0 \tag{9}$$

$$W^q_{k,f+1,5} - W^q_{k,f,2} - W^q_{k,f,4} \le 0 \tag{10}$$

And, the term $\sigma^q(t)$ has the same function as previously shown:

$$W^q_{k,f,t+L} \le Y^q_{k,f,t} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in T_{k,f}, \quad t \le T - L \tag{11}$$

Constraints (11) represent connectivity between the arrival fix and the airport. If a flight lands at the airport by time $t + L$, it must have arrived at the arrival fix by time t. In other words, the flight cannot land at the airport until it has spent L time units flying from arrival fix to airport:

$$Y^q_{k,f,t} \le Y^{\text{plan}}_{k,f,t} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in T_{k,f} \tag{12}$$

Constraints (12) ensure that the flights will not arrive at the arrival fix before the scheduled time:

$$W^q_{2,j,t+\Delta(t)^q} - W^q_{1,i,t} - S^q_{ij} \le \sigma^q(t) \quad \forall\, i, j \in \mathcal{F}\xi_q \in \mathcal{H}, \quad t \in \Gamma, \qquad t \le T - \inf\, \Delta^q \tag{13}$$

$$W^q_{1,i,t+\Delta(t)^q} - W^q_{2,j,t} + S^q_{ij} \le \sigma^q(t) + 1 \qquad \forall\, i, j \in \mathcal{F}\xi_q \in \mathcal{H}, \quad t \in \Gamma, \qquad t \le T - \inf\, \Delta^q \tag{14}$$

Constraints (13) and (14) represent connectivity between the two paths. The flights on one path obey the first–come/first-served rule, but they can change order with flights on the other path. In other words, any pair of flights $(f_i f_j)$ can reverse, f_i is any flight on path 1, and f_j is any flight on path 2. If flight f_i lands before flight f_j, then we set $S_{ij} = 0$. So, constraints (14) become redundant and constraints (13) ensure the required time separation between these two flights. Similarly, if flights f_j is landing before flight f_i, we set $S_{ij} = 1$. Then, constraints (13) become redundant and constraints (14) ensure the required time separation between these two flights.

$$Y^{\xi_q}_{k,f,t} = Y^{\xi_{q+1}}_{k,f,t} \quad \forall\, k \in K, \qquad f \in \mathcal{F}, \qquad t \in T_{k,f}, \quad q \in \{1, \ldots, Q-1\}, \qquad \xi_q \in \mathcal{H}, \qquad 1 \le t \le \tau_q \tag{15}$$

$$W^q_{k,f,t}, \qquad Y^q_{k,f,t} \in \{0, 1\} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \tag{16}$$

Constraints (15) are a set of coupling constraints (Richetta and Odoni [14]) on the decision variables of the arriving time at the arrival fix $Y^q_{k,f,t}$.

These constraints equate the specific planned arrival decisions under different scenarios, which force ground-holding decisions to be the same for all scenarios passing through the same node at that time. For example, in Fig. 1, scenarios ξ_1 and ξ_2 pass through the same nodes before the scenario tree diverges, which is from time 1 to time τ_1. So, all the decision variables of both scenarios ξ_1 and ξ_2 must be the same, which means

$$Y^1_{k,f,t} = Y^2_{k,f,t} \qquad \forall\, k \in K, \qquad f \in \mathcal{F}, \qquad 1 \le t \le \tau_1$$

And, similarly, for scenarios ξ_2 and ξ_3, we also have

$$Y^2_{k,f,t} = Y^3_{k,f,t} \quad \forall\, k \in K, \qquad f \in \mathcal{F}, \qquad 1 \le t \le \tau_2$$

Note here that scenarios ξ_1 and ξ_3 also pass through the same nodes before $t = \tau_1$, but the two previous constraints already include this relationship; there is no need to add more constraints here. Constraints (16) ensure the variable to be binary.

For simplicity, this paper only considers a single change for separation and only addresses two merging flows. However, this model could be extended to handle more complex situations. For example, in the case of a more complex separation profile with multiple changes, this model only needs the specific separation for each scenario at each time step; and this will not increase the complexity of this model. The switch key term $\sigma^q(t)$ may need to be changed to handle multiple safe separation choices. For adding more merging flows, the dimension of the reversal decision variable S_{ij} needs to be modified to express more paths. The number of reversal decision variables will increase with the number of paths. Adding more merging flows is likely to increase the complexity of this model and may require more efficient computation algorithm.

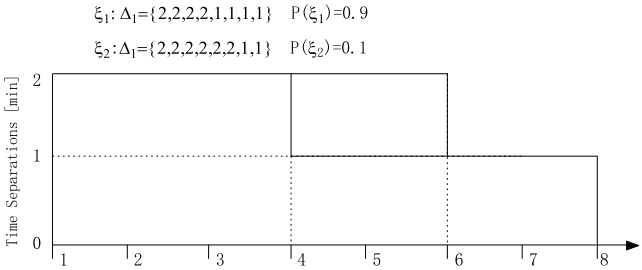

Fig. 2 Separations corresponding to different scenarios.

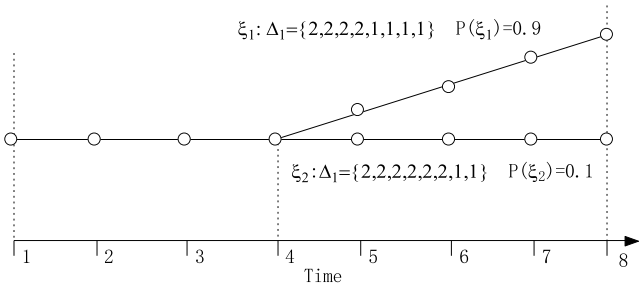

Fig. 3 Scenario tree for small example.

Moreover, our mathematical model can also handle metering operation for arrival merging flow by replacing the arrival fix with the merge point and the airport with the metering point. Delay could be absorbed relatively far from the merge point (lower cost) or relatively close to the merge point (high cost). Details are included in the Appendix.

C. Examples with Small Size Problem

To clearly illustrate the properties of our model, presented in the last section, we apply it to a small problem. We assume there are four aircraft in total, with two each on each path ($N_1 = 2$, $N_1 = 2$), and the total time period is eight; $T = 8$. Let the cost ratio between airborne holding and ground holding be two; $\lambda = 2$. The time window for each flight is set as follows: $T_{1,1} = \{1, 2\}$, $T_{1,2} = \{3, 4, 5, 6\}$, $T_{2,1} = \{1, 2, 3\}$, and $T_{2,2} = \{4, 5, 6\}$; and the first time period $\underline{T_{k,f}}$ is set as the scheduled arrival time at the arrival fix for each aircraft. We set the required flight time from the arrival fix to the airport to be two; $L = 2$. There are two separation scenarios: $H = \{\xi_1, \xi_2\}$.

As shown in Fig. 2, both of the scenarios begin with greater separation $\bar{\Delta} = 2$, which might be due to the fog in the morning. And, they will change to a small separation $\underline{\Delta} = 1$ later, when perhaps the fog disappears. The only difference between these two scenarios is the timing of the separation change. For scenario 1, the changing time is at $t = 4$. And, it is two units of time later for scenario 2 ($t = 6$). The detail for the scenario tree of our example is shown in Fig. 3.

1. *Example 1: Difference Between Uncontrolled Mode and GDP Mode*

We use scenario 1 in this deterministic example to compare the uncontrolled result and the result under the GDP. The solutions are shown in Table 1, in which GH and AH stand for ground holding and airborne holding. We find that, by implementing the GDP with a perfect weather forecast (deterministic model), all airborne holding can be replaced by ground holding. Due to the high cost of airborne holding, it is much cheaper to implement ground delay.

Table 1 Difference between uncontrolled mode and GDP mode

	Uncontrolled				GDP			
Flight	Fix	Airport	GH	AH	Fix	Airport	GH	AH
1.1	1	3	0	0	1	3	0	0
1.2	3	6	0	1	4	6	1	0
2.1	1	5	0	2	3	5	2	0
2.2	4	7	0	1	5	7	1	0
Total cost	8				4			

2. *Example 2: Difference Between Two Paths and One Single Path*

We use scenario 1 as a deterministic model to demonstrate the difference between two paths and one single path. For one single path, it obeys the first-come/first-served rule. Bayen and Tomlin [22] and Bayen et. al. [23] attempted to solve a similar problem by transferring it into a schedule problem and proving the fixed arrival order was not the optimal solution. Here, we only consider that aircraft on different paths can switch arrival order, but the arrival order is still fixed on each path.

We assume for one single path that the fixed arrival order is $f_{2.1}$, $f_{1.1}, f_{1.2}, f_{2.2}$. We apply the GDP on both situations, and the result is shown in Table 2. We can find that, for the two-path problem, the order of $f_{2.1}$, $f_{1.1}$ is switched to reduce the total cost. Instead of assigning one unit of ground delay and one unit of airborne delay to flight $f_{1.1}$, which costs $1 + 2 = 3$, the two-path problem lets flight $f_{1.1}$ arrive first and assigns two units of ground delay to flight $f_{2.1}$,which costs only two. So, the advantage of the two-path problem is that it allows aircraft on different paths to change arrival order to mitigate the total delay cost; at the same time, it still applies the first-come/first-served policy on each path to make it easy for implementation in reality.

3. *Example 3: Dynamic Stochastic Model with Different Probability Mass Functions of Scenarios*

We need to specify scenario probabilities for each scenario first before we apply our dynamic stochastic model. First, let us set $P\{\xi_1\} = 0.9$ and $P\{\xi_2\} = 0.1$, which means the first scenario has a very high probability to realize. In other words, the weather conditions will improve early ($t = 4$) with high probability. So, we will prefer to schedule the flight arrival time earlier to reduce the unnecessary ground delay. Even though this decision could risk airborne delay, the probability of airborne delay happening is very low. The results shown in Table 3 proved our aforementioned assumption. As we can see, the actual landing times of flights $f_{1.2}$ and $f_{2.2}$ are different in each scenario. For flight $f_{1.2}$, the decision is made before the scenario tree diverges, so their decision is the same (it will arrive at the arrival fix at $t = 4$). Although it could face one unit of airborne delay after it arrives at the arrival fix if scenario 2 happens, the expected cost is low. For flight $f_{2.2}$, the decision is made after the diverge time, so it can choose the best strategy to reduce the total delay, respectively, in each scenario. Similarly, if we set $P\{\xi_1\} = 0.1$ and $P\{\xi_2\} = 0.9$, which means scenario 2 has a high chance of being

Table 2 Difference between two paths and one single path

		One single path				Two paths			
Time window	Flight	Fix	Airport	GH	AH	Fix	Airport	GH	AH
{1, 2}	1.1	2	5	1	1	1	3	0	0
{3, 4, 5, 6}	1.2	4	6	1	0	4	6	1	0
{1, 2, 3}	2.1	1	3	0	0	3	5	2	0
{4, 5, 6}	2.2	5	7	1	0	5	7	1	0
Total cost			5				4		

Table 3 Stochastic model with probability mass function ($P\{\xi_1\} = 0.9, P\{\xi_2\} = 0.1$)

		Scenario 1 ($P\{\xi_1\} = 0.9$)				Scenario 2 ($P\{\xi_2\} = 0.1$)			
Time window	Flight	Fix	Airport	GH	AH	Fix	Airport	GH	AH
{1, 2}	1.1	1	3	0	0	1	3	0	0
{3, 4, 5, 6}	1.2	4	6	1	0	4	7	1	1
{1, 2, 3}	2.1	3	5	2	0	3	5	2	0
{4, 5, 6}	2.2	5	7	1	0	6	8	2	0
Total cost			4				6		
Expected total cost			3.6				0.6		

Table 4 Stochastic model with probability mass function ($P\{\xi_1\} = 0.1, P\{\xi_2\} = 0.9$)

		Scenario 1 ($P\{\xi_1\} = 0.1$)				Scenario 2 ($P\{\xi_2\} = 0.9$)			
Time window	Flight	Fix	Airport	GH	AH	Fix	Airport	GH	AH
{1, 2}	1.1	1	3	0	0	1	3	0	0
{3, 4, 5, 6}	1.2	5	7	2	0	5	7	2	0
{1, 2, 3}	2.1	3	5	2	0	3	5	2	0
{4, 5, 6}	2.2	6	8	2	0	6	8	2	0
Total cost			6				6		
Expected total cost			0.6				5.4		

realized, the weather will probably become good late ($t = 6$). As the results show in Table 4, one more unit of ground delay is assigned to flight $f_{1.2}$. It could be an unnecessary ground delay if scenario 1 happens in reality, but the expected cost is low. As a result of the conservative decision on flight $f_{1.2}$, one more ground delay is also assigned to flight $f_{2.2}$ to ensure the separation between flights. So, our dynamic stochastic model can adjust the schedule based on a different probability mass function to make the best strategy for the weather forecast at that time.

III. Dual Decomposition Method

A. Complexity of the Problem

A realistic and accurate problem is of a very large size, which is difficult to solve as a whole. The aforementioned model is a disaggregate model, and the decision variables are related to each individual flight. The number of variables is $2 \times N \times T \times Q$, where N is the number of flights, T is the time period, and Q is the scenario numbers. The variables could be up to hundreds of thousands for a busy hub airport like Hartsfield–Jackson Atlanta International Airport (ATL). For example, a 2 h problem involves approximately 100 flights and four scenarios. Then, there are $120 \times 100 \times 4 = 48,000$ landing variables $W^q_{k,f,t}$ and the same for variable $Y^q_{k,f,t}$. So, the number of decision variable is up to 96,000. Moreover, the reversal decision variables S^q_{ij} should be considered. So, the total number of variable could be more than 100,000. For such a large-sized integer program, even the most up-to-date optimization solver cannot solve it in a reasonable time [24]. However, all constraints are separate for each scenario, except for the coupling constraints [Eq. (9)]. In large-scale optimization, the dual decomposition method is often used to separate the problem into several small problems.

The dual decomposition method was first proposed by Dantzig and Wolfe to solve large-scale problems [25]. More recently, Sun et al. [26], Cao and Sun [27], and Rios and Ross [28] used the dual decomposition method to tackle the arrival scheduling problem, which is known to be NP hard. By using dual decomposition method, each scenario becomes a smaller subproblem, which can be solved separately or even in parallel. Note that, even though our scenario number is not very large, the solution time is not linear to the problem size. Solving each scenario separately is much faster than solving them as a whole. For a large-scale problem, the difference could even be whether this problem can be solved or not.

B. Dual-Problem Formulation

Step 1: Decompose the terms scenario by scenario; the objective function is a summation of the total delay of each scenario.
We define

$$f^q(Y^q, W^q) = P\{q\}\left\{\sum_{k\in K}\sum_{f\in\mathcal{F}}\left(\sum_{t\in T_{k,f}}(Y^{\text{plan}}_{k,f,t} - Y^q_{k,f,t})\right) + \lambda\left(\sum_{k\in K}\sum_{f\in\mathcal{F}}\left(\sum_{t\in T_{k,f}}(Y^q_{k,f,t} - W^q_{k,f,t+L})\right)\right)\right\} \tag{17}$$

So, the objective function can be rewritten as follows:

$$\text{Min}\sum_{\xi_q\in\mathcal{H}} f^q(Y^q, W^q) \tag{18}$$

Step 2: By forming the partial Lagrangian for the last constraints (coupling constraints), we can obtain the dual problem

$$g(v) := \text{Max}_{v\geq 0}\text{Min}\sum_{\xi_q\in\mathcal{H}} f^q(Y^q, W^q) + \sum_{q=1}^{Q-1}\sum_{k\in K}\sum_{f\in\mathcal{F}}\sum_{t=1}^{\tau_q}(v^q_{k,f,t}(Y^{\xi_q}_{k,f,t} - Y^{\xi_{q+1}}_{k,f,t})) \tag{19}$$

such that

$$W^q_{k,f,t} \leq W^q_{k,f,t+1} \quad \forall\, k\in K, \qquad f\in\mathcal{F}\xi_q\in\mathcal{H}, \qquad t\in\Gamma \tag{20}$$

$$Y^q_{k,f,t} \leq Y^q_{k,f,t+1} \quad \forall\, k\in K, \qquad f\in\mathcal{F}\xi_q\in\mathcal{H}, \qquad t\in\Gamma \tag{21}$$

$$W^q_{k,f+1,t+\Delta(t)^q} - W^q_{k,f,t} \leq \sigma^q(t) \quad \forall\, k\in K, \qquad f\in\mathcal{F}\xi_q\in\mathcal{H}, \quad t\in\Gamma, \qquad t\leq T - \inf\Delta^q \tag{22}$$

where

$$\sigma^q(t) = \begin{cases} W^q_{k,f,t+(\bar{\Delta}-\underline{\Delta})} & t\in[\tau_q, \tau_q - (\bar{\Delta}-\underline{\Delta}) + 1] \\ W^q_{k,f,t-(\bar{\Delta}-\underline{\Delta})} & t\in[\tau_q + 1, \tau_q + (\bar{\Delta}-\underline{\Delta})] \\ 0 & \text{otherwise} \end{cases}$$

$$W^q_{k,f,t+L} \leq Y^q_{k,f,t} \quad \forall\, k\in K, \quad f\in\mathcal{F}\xi_q\in\mathcal{H}, \qquad t\in T_{k,f}, \qquad t\leq T - L \tag{23}$$

$$Y^q_{k,f,t} \leq Y^{\text{plan}}_{k,f,t} \quad \forall\, k\in K, \qquad f\in\mathcal{F}\xi_q\in\mathcal{H}, \qquad t\in T_{k,f} \tag{24}$$

$$W^q_{2,j,t+\Delta(t)^q} - W^q_{1,i,t} - S^q_{ij} \leq \sigma^q(t) \quad \forall\, i,j\in\mathcal{F}\xi_q\in\mathcal{H}, \quad t\in\Gamma, \qquad t\leq T - \inf\Delta^q \tag{25}$$

$$W^q_{1,i,t+\Delta(t)^q} - W^q_{2,j,t} + S^q_{ij} \leq \sigma^q(t) + 1 \quad \forall\, i,j\in\mathcal{F}\xi_q\in\mathcal{H}, \quad t\in\Gamma, \qquad t\leq T - \inf\Delta^q \tag{26}$$

$$W^q_{k,f,t}, \qquad Y^q_{k,f,t}\in\{0,1\} \quad \forall\, k\in K, \qquad f\in\mathcal{F}\xi_q\in\mathcal{H}, \qquad t\in\Gamma \tag{27}$$

Step 3: Combine the coupling constraints that belong to the same scenario. Use array v_q to express the Lagrange multiplier for each scenario and use arrays Y^q and W^q to express the corresponding decision variables under the same scenario. Then, rearranging the terms in the objective function of the dual problem to group the terms scenario by scenario, we can obtain the master problem

$$g^* = \text{Max}_{v\geq 0}\sum_{q=1}^{Q} g^q(v_1, v_2, \ldots, v_{Q-1}) \tag{28}$$

where

$$g^q(v_1, v_2, \ldots, v_{Q-1}) = \min\left(f^q(Y^q, W^q) + \sum_{j=1}^{Q-1} v_j^T Y^q\right) \quad (29)$$

which is the subproblem for each scenario q.
Note that

$$v_q = [v^q_{k,f,t}]_{k\in K, f\in\mathcal{F}, t\in\Gamma} Y_q = [Y^q_{k,f,t}]_{k\in K, f\in\mathcal{F}, t\in\Gamma} W_q$$
$$= [W^q_{k,f,t}]_{k\in K, f\in\mathcal{F}, t\in\Gamma}$$

is the array expression for simplicity.

Step 4: The iterations are as follows:
Subproblem:

$$g^q(v_1, v_2, \ldots, v_{Q-1}) = \min\left(f^q + \sum_{j=1}^{Q-1} v_j^T Y^q\right),$$
$$q \in \{1, \ldots, Q\} \quad (30)$$

such that

$$W^q_{k,f,t} \le W^q_{k,f,t+1} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \quad (3.15)$$

$$Y^q_{k,f,t} \le Y^q_{k,f,t+1} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \quad (31)$$

$$W^q_{k,f+1,t+\Delta(t)^q} - W^q_{k,f,t} \le \sigma^q(t) \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H},$$
$$t \in \Gamma, \qquad t \le T - \inf \Delta^q \quad (32)$$

where

$$\sigma^q(t) = \begin{cases} W^q_{k,f,t+(\bar{\Delta}-\underline{\Delta})} & t \in [\tau_q, \tau_q - (\bar{\Delta} - \underline{\Delta}) + 1] \\ W^q_{k,f,t-(\bar{\Delta}-\underline{\Delta})} & t \in [\tau_q + 1, \tau_q + (\bar{\Delta} - \underline{\Delta})] \\ 0 & \text{otherwise} \end{cases}$$

$$W^q_{k,f,t+L} \le Y^q_{k,f,t} \quad \forall\, k \in K,$$
$$f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in T_{k,f}, \qquad t \le T - L \quad (33)$$

$$Y^q_{k,f,t} \le Y^{\text{plan}}_{k,f,t} \quad \forall k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in T_{k,f} \quad (34)$$

$$W^q_{2,j,t+\Delta(t)^q} - W^q_{1,i,t} - S^q_{ij} \le \sigma^q(t) \quad \forall\, i, j \in \mathcal{F}\xi_q \in \mathcal{H},$$
$$t \in \Gamma, \qquad t \le T - \inf \Delta^q \quad (35)$$

$$W^q_{1,i,t+\Delta(t)^q} - W^q_{2,j,t} + S^q_{ij} \le \sigma^q(t) + 1 \quad \forall\, i, j \in \mathcal{F}\xi_q \in \mathcal{H},$$
$$t \in \Gamma, \qquad t \le T - \inf \Delta^q \quad (36)$$

$$W^q_{k,f,t}, Y^q_{k,f,t} \in \{0, 1\} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \quad (37)$$

The dual decomposition algorithm flowchart is shown in Fig. 4. The whole problem is decomposed scenario by scenario. Each subproblem is an independent optimization problem, which is easier to solve. To solve the dual problem, we need to compute the subgradient of the dual function and update the Lagrange multiplier and step size of each loop. The subproblems can be solved in parallel to reduce computation time.

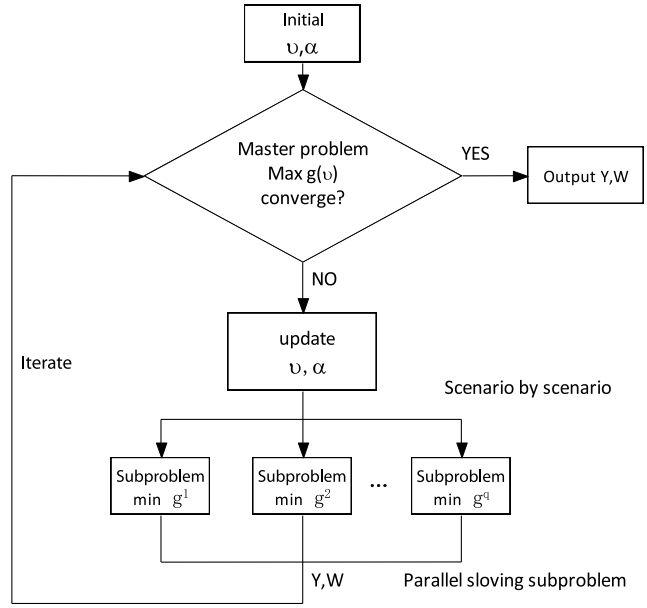

Fig. 4 Flowchart for dual decomposition algorithm.

C. Computing Improvement

To demonstrate the computing improvement by using the dual decomposition method, a half-hour case is studied that has 10 flights on each path and three scenarios in total. Based on our inputs, the experiment problem has 25,370 constraints and 3988 decision variables. The solution time is sensitive to the parameters of input. The model was solved 10 times, and the average solving time was 893 s. However, after the problem was decomposed scenario by scenario, each scenario was a subproblem for which the solution time is much shorter. The average solution times for the three scenarios were around 0.2011, 0.1926, and 1.2742 s. On average, the objective value converged in 14 steps. This means the total computing time by the dual decomposition method was 38 times faster. Moreover, if we consider solving the subproblems in parallel, then the computing time could be up to 50 times faster.

Since the computing time is not linearly related to the problem size, the dual decomposition method with parallel computing will reduce the computation time. More important, the unsolvable large-sized problem can be converted into several solvable subproblems, and it can be solved step by step. This is the key advantage of the dual decomposition method.

IV. Experiments with Large-Scale Problem

A. Experimental Setup

Now, we consider a large-scale problem with many more flights and a longer planning time. The arrival schedule between 1100 to 1200 hrs on 13 October 2013 at Hartsfield–Jackson Atlanta International Airport, taken from The Bureau of Transportation Statistics database[¶] is used in our experiment, shown in Table 5.

The model was programmed with C++ as a single-thread program on a 2.8 GHz INTEL i7 CPU, with a 16 GB RAM Dell workstation running LINUX. The mathematical programming solver Gurobi5.6.3 was used, which is capable of solving the IP problem [29].

We will study the benefit of using our model in this section. First, we build a baseline case by our model and an alternative case in which the arrival order is fixed. Through comparing the results of the baseline case with the result of the alternative case and the best result by assuming perfect information, we can get interesting insight from our model.

[¶]Data available online at www.bts.gov [retrieved 10 October 2014].

Table 5 Arrival schedule between 1100 and 1200 hrs on 13 October 2013 at ATL

Path 1: total 22 flights, hrs				Path 2: total 18 flights, hrs			
1105	1107	1109	1117	1102	1104	1107	1111
1123	1127	1128	1130	1112	1113	1114	1120
1135	1136	1137	1138	1125	1126	1128	1132
1140	1143	1146	1150	1137	1138	1139	1151
1151	1152	1154	1157	1157	1158	— —	— —
1158	1159	— —	— —	— —	— —	— —	— —

Table 6 Parameter settings for baseline case

Parameter	Value
N_1	22
N_2	18
T/ min	90
λ	2
L/ min	10
Scenarios	3
$\bar{\Delta}$/ min	5
$\underline{\Delta}$/ min	1

1. Case 1: Baseline Case

We consider a total of 40 flights scheduled to arrive at ATL between 1100 to 1200 hrs (Table 5). As shown in Table 6, there are 22 scheduled flights on path 1 and 18 scheduled flights on path 2. The two merging flows are from the northwest and southwest. The window slot duration is random, assigned between 20 and 30 min. We will set $T = 90$ min, and we extend to a half-hour in case some flights may face longer delays. Note here that the problem size is related to the planning time periods, so it is critical to choose a proper value for T. A large T value will make the problem size too large to solve, but a too small T value may bring the situation that not all flights have landed. Let the cost ratio between the airborne holding and ground holding be two ($\lambda = 2$) for the baseline case. We assume the required flight time from the arrival fixes to the airport is 10 min, which means $L = 10$.

We will consider three time-separation scenarios: $\mathcal{H} = \{\xi_1, \xi_2, \xi_3\}$. In each of them, we assume weather only changes once: from bad weather (big separation $\bar{\Delta} = 5$ min) to good weather (small separation $\underline{\Delta} = 1$ min). But, the exact timing of the weather changing depends on the scenario. For the first scenario, the weather will become good starting at 1110 hrs, inducing a small separation. The changes in the second scenario and third scenario will happen at 1120 and 1130 hrs, respectively.

Figure 5 shows the scenario tree of our case 1. We can see that, at 1110 and 1120 hrs, two new branches come out, corresponding to different scenarios. Each scenario occurs with a probability. Here, we set the probability mass function for case 1 as follows:

Table 7 Expected delay costs for all cases[a]

	Ground delay cost	Airborne delay cost	Total delay cost
Deterministic case	42.8	0.0	42.8
Case 1	43.7	9.8	53.5
Case 2	53.3	14.2	67.5

[a]Note that the airborne-delay cost is already multiplied by the cost ratio in this table.

$$P\{\xi_1\} = 0.8; \qquad P\{\xi_2\} = 0.1; \qquad P\{\xi_3\} = 0.1$$

which means that the first scenario will happen with a high chance; we expect to observe early arrival decisions and less ground hold.

2. Case II: Fix the Arrival Order of Flights

In this case, we demonstrate our model's ability to reduce delay by allowing flights to switch arrival order with other flights on the other path. We fixed the arrival order based on the original schedule, shown in Table 5. So, it is equal to a single-path problem with a fixed arrival order, all flights obey the first-come/first-served rule.

3. Deterministic Case: Assuming Perfect Information

Besides the two cases, a "perfect information" case (deterministic case) is calculated to work as an ideal case. For the deterministic case, information is perfect for each scenario, which means we can assign ground delays to replace the airborne delays. We calculated the schedule for each scenario separately, accounting for its specific deterministic separation profile. Then, they are multiplied with their associated scenario probabilities to get the ideal delay cost. And, we compare the results of the four stochastic cases with the deterministic case to measure the total delay in percentage.

B. Results

Table 7 summarizes the expected delay results across the cases. In case 1, we compare our stochastic model with the deterministic model. In our stochastic model, about 25% more delays are assigned, especially some airborne delays among them. For the deterministic case, information is perfect for each scenario, which means we can assign ground delays to replace the airborne delays. For example, if we know a flight will face an airborne delay for four units of time after it arrives at the arrival fix, we can assign four more units time to the ground delays to make sure this flight will not wait when it approaches the airport. But, for the stochastic case, the information about future weather conditions is not perfect; each scenario has a chance to occur, which may cause airborne delays. For example, the first scenario will happen with a high chance in case 1. Most flights will be assigned less ground delays to arrive at the fix as the scheduled plan due to the high probability for good weather to occur at 1110 hrs. The detail of each flight's schedule is shown in Figs. 6 and 7. We can find that most decisions coincide with the scheduled plan before the

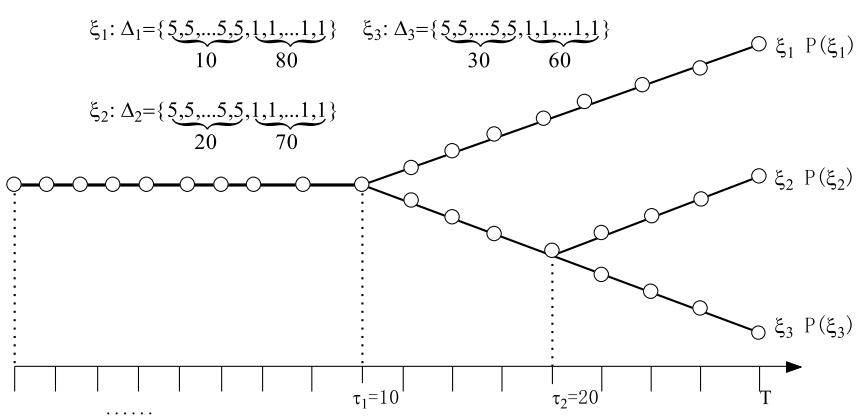

Fig. 5 Scenario tree for case 1.

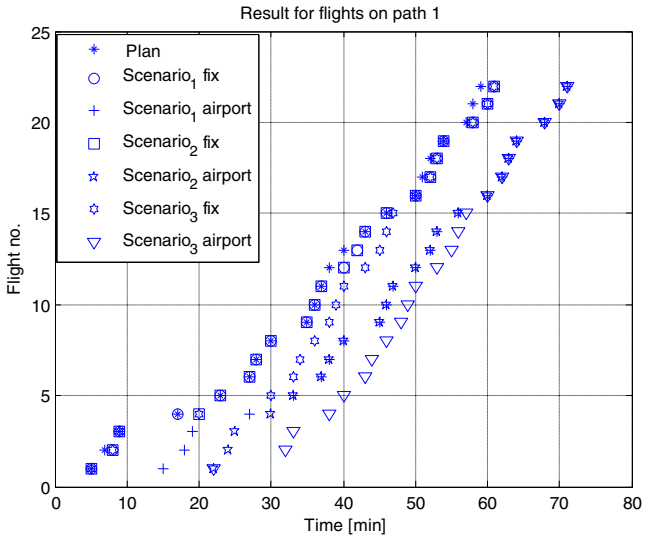

Fig. 6 Schedule result for flights on path 1 in case 1.

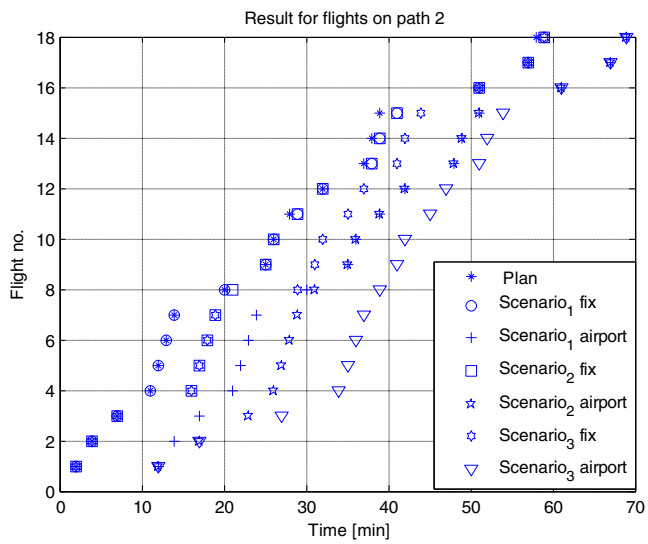

Fig. 7 Schedule result for flights on path 2 in case 1.

scenario "tree's" diverging point ($t = 10$). Flights will face airborne delays if the second scenario and the third scenario happen, but their probabilities are very low relative to the first scenario. So, the expected total delay cost is optimal even there are airborne delays here. On the other hand, our delay moderate cost ratio ($\lambda = 2$) also contributes to this result.

Another difference between the deterministic case and the stochastic case is that all three scenarios are calculated separately as three small problems in the deterministic case; but, for the stochastic case, all three scenarios are solved together as one problem. As a result of the coupling constraint, all decisions are the same before the scenario tree's diverging point because the decision can only be made on information available at that time. In Figs. 6 and 7, we can see that all three scenarios' arrival decisions at the fix are the same before the first scenario tree's diverging point ($t = 10$). And, the second scenario's arrival decisions at the fix are also the same with the third scenario before they diverge ($t = 20$). This feature makes the stochastic case have more delays because it cannot adjust decisions separately for each scenario, whereas the deterministic case with the perfect information can.

The result from case 2 demonstrates our model's ability to reduce the expected delay cost when compared with the fixed arrival order schedule. The fixed arrival order case is equal to a single-path problem in which all flights obey the first-come/first-served rule. Adding one more path is like giving a little more freedom to the optimization problem to find a better solution. The performance improvement is not much by our two-path model in this experiment; it only reduces the expected delay cost by 20%. This is due to our loose schedule plan. With a tighter schedule with many overlaps in the scheduled arrival time period, our model will reduce the delay cost much more. We can also try to give more freedom to the problem, such as allowing all of the flights to switch orders with each other. Under this assumption, we can expect a more optimal schedule with much less delay cost, but the complexity of the problem will also increase significantly, which possibly cannot be handled.

In addition, the solution time of the original problem is long according to our experimentations. Based on our inputs, the experiment problem size is large, with 254,570 constraints and 22,788 decision variables. The solution time can be up to 200,000 s, and it is sensitive to the parameters of input. In most cases, the problem was not solved, even after 200,000 s. After the problem is decomposed scenario by scenario, each scenario is a subproblem, for which the solution time is much shorter. The average solution times for the three scenarios are 238, 975, and 8742 s. On average, the objective value will converge in 17 steps. Moreover, if we solve the subproblem in parallel, the computing time could be around $8742\ \mathrm{s} \times 17 = 148,614$ s. The solution time is still long, but we converted an unsolvable large-sized problem into a solvable one. This is the critical improvement by the dual decomposition method.

In summary, our experiments suggest that the expected delay is strongly related to how soon the new information becomes available after the beginning of the planning period. From case 1 and case 2, we can see different problem instances affect the result a lot. A more accurate forecast and quicker updating information are the keys to reducing the delays. Moreover, a flexible arrival order is preferred over a fixed one, especially for a higher schedule with significant overlaps in the scheduled arrival time period.

V. Conclusions

This paper presents a dynamic stochastic 0–1 IP model for converging inbound air traffic. The mathematical analysis and the experimental results show that this model can overcome the limitation in which the individual flight cannot be scheduled very accurately in previous models because the basic period of time of the AAR is normally 15 min or more. This model also addresses a problem involving the merging of two inbound streams into a single airport in which the uncertainty of separation from the minutes-in-trail restriction is considered. The simulation experiment has shown that allowing flights on different paths to switch arrival orders will help reduce the total delays. Ideally, this model can be extended to perform with more freedom on the arrival order. However, the complexity of the problem increases very quickly as more freedom is given to the flight's arrival order.

Since the stochastic model is built on the scenario tree method, the optimization problem can be decomposed scenario by scenario via a dual decomposition method. Taking advantage of multicore computers, the computational efficiency is further improved by solving the subproblems in parallel.

Appendix A: Metering Operation Model for Arrival Merging Flow

Our mathematical model can also handle metering operations for arrival merging flows as they approach a merge point before landing. Delays could be absorbed relatively far from the merge point (at a lower cost) or relatively close to the merge point (at a higher cost). As

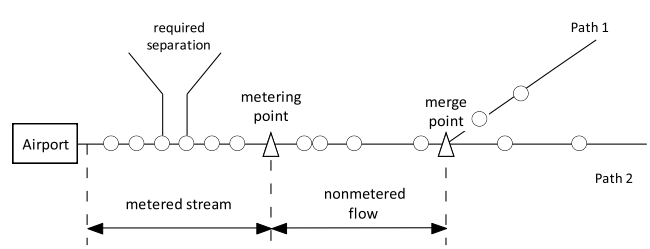

Fig. A1 Metering operation for arrival merging flow.

shown in Fig. A1, two sets of flights are scheduled to merge into one flow before landing [30]. Metering constraints take effect after the metering point to meet restrictions imposed at the destination airport. The required separation between flights in the metered stream is time varying and different from scenario to scenario. Without loss of generality, we may also use $\Delta(t)^q$ to denote the specific time separation. The airspace between the merge point and the metering point is used to adjust aircraft for metering, such as slowing down and/or airborne holding.

Meanwhile, we may use the same set of notations in this model, whereas the notations now represent slightly different practical meanings, as explained in the following. The required flight time from the merge point to the metering point is donated by L. Let λ denote the cost ratio between one unit of airborne delay before and after the merge point. We consider $\lambda > 1$ because the airborne delay after the merge point is more expensive, and we assume it is the same for all flights.

A1. Decision Variables Redefinition

The decision variables in the model are binary variables defined as follows:

$$W^q_{k,f,t} = \begin{cases} 1 & \text{if flight } f \text{on path} k \text{ is arrives at metering point by time } t \text{ under scenario } \xi_q \\ 0 & \text{otherwise} \end{cases}$$

$$Y^q_{k,f,t} = \begin{cases} 1 & \text{if flight } f \text{ on path} k \text{ arrives at merge point by time } t \text{ under scenario } \xi_q \\ 0 & \text{otherwise} \end{cases}$$

Note that only the physical meaning of the decision variables has been changed.

A2. Objective Function Redefinition

The objective of the model is to minimize the expected combination cost of the airborne-holding delay for all flights:

$$\text{Min} \sum_{\xi_q \in \mathcal{H}} P\{q\} \left\{ \sum_{k \in K} \sum_{f \in \mathcal{F}} \left(\sum_{t \in T_{k,f}} (Y^{\text{plan}}_{k,f,t} - Y^q_{k,f,t}) \right) + \lambda \left(\sum_{k \in K} \sum_{f \in \mathcal{F}} \left(\sum_{t \in T_{k,f}} (Y^q_{k,f,t} - W^q_{k,f,t+L}) \right) \right) \right\} \tag{A1}$$

where

$$Y^{\text{plan}}_{k,f,t} = \begin{cases} 1 & \text{if flight } f \text{ on path } k \text{ is planned to arrive at merge point by time } t \\ 0 & \text{otherwise} \end{cases}$$

Let $Y^{\text{plan}}_{k,f,t}$ denote the corresponding binary values of the scheduled arrival plan at the merge point, which is the prior information for this model and is deterministic and the same for all scenarios. In Eq. (A1), the first component is the difference between the scheduled arrival time and the actual arrival time at the merge point, which expresses the low-cost airborne delay. The second component is the difference between the scheduled arrival time and the actual arrival time at the metering point, which is the high-cost airborne holding delay. The cost ratio λ is a multiplier that increases the penalty for airborne delay after the merging point.

A3. Constraints

All constraints have the same formulation as the original model. The only difference is the meanings of variables. For examples, W is changed from landing at the airport to arriving at the metering point, and the meaning of variable Y is changed from arriving at the arrival fix to arriving at the merge point. The constraints are shown as follows:

$$W^q_{k,f,t} \le W^q_{k,f,t+1} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \tag{A2}$$

$$Y^q_{k,f,t} \le Y^q_{k,f,t+1} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \tag{A3}$$

$$W^q_{k,f+1,t+\Delta(t)^q} - W^q_{k,f,t} \le \sigma^q(t) \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \quad t \in \Gamma, \qquad t \le T - \inf \Delta^q \tag{A4}$$

where

$$\sigma^q(t) = \begin{cases} W^q_{k,f,t+(\bar{\Delta}-\underline{\Delta})} & t \in [\tau_q, \tau_q - (\bar{\Delta} - \underline{\Delta}) + 1] \\ W^q_{k,f,t-(\bar{\Delta}-\underline{\Delta})} & t \in [\tau_q + 1, \tau_q + (\bar{\Delta} - \underline{\Delta})] \\ 0 & \text{otherwise} \end{cases}$$

$$W^q_{k,f,t+L} \le Y^q_{k,f,t} \quad \forall\, k \in K, \quad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in T_{k,f}, \qquad t \le T - L \tag{A5}$$

$$Y^q_{k,f,t} \le Y^{\text{plan}}_{k,f,t} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in T_{k,f} \tag{A6}$$

$$W^q_{2,j,t+\Delta(t)^q} - W^q_{1,i,t} - S^q_{ij} \le \sigma^q(t) \quad \forall\, i, j \in \mathcal{F}\xi_q \in \mathcal{H}, \quad t \in \Gamma, \qquad t \le T - \inf \Delta^q \tag{A7}$$

$$W^q_{1,i,t+\Delta(t)^q} - W^q_{2,j,t} + S^q_{ij} \le \sigma^q(t) + 1 \quad \forall\, i, j \in \mathcal{F}\xi_q \in \mathcal{H}, \quad t \in \Gamma, \qquad t \le T - \inf \Delta^q \tag{A8}$$

$$Y^{\xi_q}_{k,f,t} = Y^{\xi_{q+1}}_{k,f,t} \quad \forall\, k \in K, \qquad f \in \mathcal{F}, \qquad t \in T_{k,f}, q \in \{1, \ldots, Q-1\}, \quad \xi_q \in \mathcal{H}, \qquad 1 \le t \le \tau_q \tag{A9}$$

$$W^q_{k,f,t}, \qquad Y^q_{k,f,t} \in \{0, 1\} \quad \forall\, k \in K, \qquad f \in \mathcal{F}\xi_q \in \mathcal{H}, \qquad t \in \Gamma \tag{A10}$$

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