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The probabilistic vehicle routing problem with service guarantees

Lijian Chen[a,*], Wen-Chyuan Chiang[b], Robert Russell[b], Jun Chen[c], Dengfeng Sun[c]

[a] Department of MIS, Operations Management, and Decision Sciences, University of Dayton, United States
[b] Finance, Operations Management and International Business, University of Tulsa, United States
[c] School of Aeronautics and Astronautics, Purdue University, United States

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ABSTRACT

We develop a two-phase approach to solving the capacitated routing problem (CVRP) with stochastic demand. A nonlinear chance-constrained optimization model is solved to determine delivery quantities, and a tabu search metaheuristic is used to determine vehicle routes. The goal of this research is to assure that a logistics company would satisfy the demands of customers with a high probability, while minimizing the overall transportation cost. We introduce the concept of premium customers, who are guaranteed a higher level of service. We show that our chance-constrained method has some strategic advantages over the CVRP with recourse approach. We examine the possibility of the logistics company charging customers selectively with an additional service fee to assure a high level of service. Moreover, we provide managerial insight on when the best time is to pay for the premium membership. We present computational results on commonly studied small to large-scale CVRP instances. A simulation study is conducted to explore the performance of the proposed chance-constrained approach using the CVRP with recourse. We conclude that our chance-constrained CVRP model could serve a logistics company well when resource costs and service guarantees are of concern.

1. Introduction

The capacitated vehicle routing problem involves finding the routes and shipping volumes for a fleet of identical, but fixed capacity vehicles to distribute a single product from a central depot or warehouse to customer locations so as to meet their periodical demands. The objective is to design a priori set of routes over a specified planning horizon in order to minimize the vehicle fleet and travel costs. The problem formulation would be a mixed integer program, in which the shipping volumes by customers could be real values. One classic paper by Anily and Awi (1990) examines distribution systems with a depot and multiple retailers, each of which faces deterministic demands. Goods are distributed to the retailers by a fleet of capacitated vehicles through efficient routes. There are many mature solution techniques, along with problem-oriented heuristics in obtaining a timely solution. When the demands of customers are random variables, the problem becomes a stochastic capacitated vehicle routing problem, and we need to determine how to handle the case when the demands on a route might exceed the planned volumes. When the needs for the goods are significant, there is a possibility that shortages may occur at some customer locations, with severe consequences. Thus, we need to develop further actions on planning routes and shipping volumes.

One further action is to use a backup shipping fleet, if available, to ship safety inventory through reloading routes to the customers with shortages. Usually the cost of maintaining a backup fleet and safety inventory would be more expensive, and the reloading

* Corresponding author.
E-mail addresses: lchenl@udayton.edu (L. Chen), wen-chyuan-chiang@utulsa.edu (W.-C. Chiang), rrussell@utulsa.edu (R. Russell), chenl241@purdue.edu (J. Chen), dsun@purdue.edu (D. Sun).

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routes, or the recourse routes, are also more costly than the planned routes. Such an action is achieved by a recourse model, in which a few reloading trips will be arranged if the demands exceed the planned shipping volumes. The model formulation is a two-stage stochastic programming with recourse, which requires the distributional information of customer demands to be accessible. The objective is to minimize the expected fleet and travel costs. Although the assumption on the accessibility of the distributional information is restrictive, there are many existing results based on recourse models (see Bertsimas, 1992; Laporte et al., 2002; Mendoza et al., 2016). A recourse model may not be the desired routing plan for most logistics companies because of the possibly large number of on-demand reloading trips, along with additional travel distance incurred. A logistics company may prefer a routing plan that satisfies customer demands with a very high probability and is subject to minimum on-demand changes.

In this paper, we study a capacitated vehicle routing model with a service guarantee. We impose a probabilistic constraint on the event of any potential shortage on a group of customers. This method leads to the decision on routing and shipping volume with a low probability of shortage and a service guarantee. We show that the chance-constrained method has strategic advantages over the two-stage stochastic approach with recourse. This method will also be more implementable due to avoiding complex recourse actions. The model is rather straightforward to managers and customers without many restrictive assumptions, despite the fact that the solution technique may be very technical.

The capacitated vehicle routing (CVRP) model with service guarantees is the vehicle routing problem with stochastic demands (VRPSD), as mentioned in Bertsimas (1988). Recently, Gendreau et al. (2016) present a seminal paper reviewing stochastic vehicle routing problems, including VRPSD, and points out several potential research directions. VRPSD is one of the most studied models among stochastic vehicle routing problems. The most popular approach has been the recourse approach. The recourse action involves taking the vehicle back to the depot to replenish upon failure and returning to the originally planned route at the point of failure. Naturally, the extra trip requires an extra cost added to the calculation of the total cost.

Gendreau et al. (1995) proposed the first optimal algorithm for VRPSD with classical recourse. The problem is solved in two stages. Whenever the vehicle capacity is attained or exceeded, the vehicle returns to the depot and resumes its collections along the planned route. The problem involves designing a first-stage solution in order to minimize the expected total cost of the second-stage solution. The problem is solved optimally using a general branch-and-cut procedure called an integer L-shaped method. Laporte et al. (2002) again proposed an integer L-shaped algorithm, enhanced with the lower bounding functions of Hjorring and Holt (1999), for the vehicle routing problem with stochastic demands, where the objective is to minimize the expected solution cost under the restriction that the expected demand of a route never exceeds vehicle capacity. They solved optimality problem instances for up to 100 customers and 4 vehicles, where demands follow a Poisson or normal distribution. Christiansen and Jens (2007) and Gauvin et al. (2014) also solved VRPSD with recourse using a branch-and-bound algorithm. More recently, Biesinger et al. (2016) used an integer L-shaped algorithm for the generalized VRPSD with preventive restocking, an alternative to the classical recourse policy. The preventive stocking strategy specifies that the vehicle can make a return trip to the depot, even before an actual stockout occurs, and therefore saves travel time and cost.

Readers are strongly encouraged to refer to Gendreau et al. (2016) for a comprehensive review of VRPSD variants and their solution methods. As mentioned previously, we adopt a different approach. We model the capacitated vehicle routing (CVRP) model with service guarantees using chance constraints. The chance constraint approach was first introduced by Charnes and Cooper (1959) in the format of an inventory management problem in which probabilistic measures were imposed individually on each constraint. The paper considers terminal tankage facilities supplied by a refinery and pickup by tankers in which the oil inventory and production, more than the usage, must be maintained at prescribed levels of probability. Following the same approach, Stewart and Golden (1983) present a pioneering model to identify the minimum cost tours subject to a threshold constraint on the probability of trip failure. A similar approach can be found in Dror et al. (1989), in which a chance constraint is imposed to ensure with an at least $1-\alpha$ probability that all customer demands on the same vehicle route does not exceed the capacity of the vehicle. They both show that under certain assumptions, the capacitated vehicle routing problem with stochastic demands can be transformed into an equivalent deterministic problem. The chance constraint model ensures that the probability of satisfying the demand of each customer will be at least $1-\alpha$. The chance constraint model in Sungur et al. (2008) is the deterministic counterpart of the problem in Laporte et al. (1985). Branda (2012) uses chance constraints to model random demand and travel times. Gounaris et al. (2013) study CVRP by comparing several model formulations. In the paper, the authors compare different stochastic vehicle routing models, including the chance constraint model and recourse models. This model is an instance of CVRP with modified demand data.

The technical difficulties associated with the chance constraint have constrained its business implementation for decades, since the chance constrained optimization was originally raised in the 1950s. The primary difficulty involves the loss of convexity, in both the feasible region and the constraint function for a chance constrained optimization in the general form. There are approaches to assuming the joint normally distributed random vectors and the transformation of the chance constrained optimization to a deterministic nonlinear program for solutions. A breakthrough to solving the non-convexity issue is to assume that the distribution of the random vector is log-concave so that both the feasible region and the constraint, with an equivalent transformation, will be convex. Even if convexity is not an issue, the second difficulty involves efficiently calculating the gradient or sub-gradient for the optimization algorithm to advance.

In this paper, we adopt a newly developed method, based on Monte Carlo and polynomial approximation to address the technical difficulties in Chen (2017) and Chen et al. (2017). The method has two advantages. First, it does not require independence among the components of the random vector. That is, customer demands are correlated rather than assumed to be independent. As long as the distribution is accessible, the method will be able to advance by iterations. This advantage broadens the perspective of this method because the random components are correlated in most chance constrained optimization problems. Second, the method will terminate within a bounded number of iterations and will efficiently calculate the gradient at each iteration. The computational cost becomes less of a concern in comparison to other approaches targeting chance constrained optimization problems.

We solve the capacitated vehicle routing problems in two phases due to the presence of integer variables for the routing decision. The first phase involves solving the chance-constrained optimization to find a supply level for all customers to maintain a service guarantee. Our method is closer to reality because the customer demands of the chance-constrained optimization are correlated. When the shipping volumes by customers are available, we use the obtained shipping volume decision as the deterministic demand and transform the original problem into a deterministic capacitated vehicle routing problem. We solve the deterministic model by either the integer programming solver or a well-chosen metaheuristic for optimal or satisfactory routing decisions. We note that the chance constrained optimization does not significantly raise the computational load. In our numerical study, the chance constrained optimization phase reaches the optimal solutions as the shipping volumes for as many as 261 customers with a service guarantee within hours. The deterministic counterpart of 261 customers for routing decisions, however, may take a long time to reach the true optimal solution due to the explosion regarding the number of integer variables.

One of the primary contributions of this research is to highlight the strategic advantage of the chance-constrained CVRP. We solve three models: the chance-constrained CVRP without recourse, the chance-constrained CVRP with recourse, and CVRP with recourse, on nine capacitated vehicle routing problems with varying network scales, ranging from 31 to 261 customers, and the same level of demand uncertainty. We need the chance-constrained CVRP with recourse because at least theoretically, the chance-constrained CVRP would only guarantee a service level that is less than 100%, while CVRP with recourse will satisfy customer demands with total satisfaction by arranging reloading trips. Our simulation study demonstrates that the chance-constrained CVRP has some strategic advantages over CVRP with recourse and that these methods are quite different in many ways. The routing plan of the chance-constrained CVRP is fixed over the time simulated, and nearly no reloading trips are necessary. In the recourse method, however, reloading trips are a primary means of satisfying all customer demands, and the number of reloading trips exhibits large variations, as well as the total travel distance. In practice, this notation implies that drivers, under the recourse model, may have to work overtime or deliver on the second day due to the driver's working hour limitations.

We also compare our chance-constrained CVRP model with a 100% delivery guarantee policy for high-profile customers, which offers a similar alternative to our model by routing the vehicle to visit the high-profile, i.e., premium customers first. We note that in one of our numerical results of 10,000 simulation trials, there were never any violations of the route obtained by the chance-constrained CVRP method for premium customers. The comparison of the chance-constrained CVRP and the 100% delivery guarantee policy suggests that the chance-constrained CVRP would offer satisfactory performance to the logistics company as a comprehensive decision support tool regarding the service guarantee and demand uncertainty.

The last primary contribution is the managerial implications on the pricing of premium customers. Through the numerical results, we find that imposing the chance constraint to achieve a high service guarantee, such as 95% on all customers, would significantly increase the overall cost so as to be unrealistic. However, if we only impose the chance constraint on a small group of customers, premium customers, the cost increase will be rather moderate and acceptable. In such a model, we meet the non-premium customers' demands with the shipping volume of their mean demands without considering any variations. For premium customers, however, we satisfy their demands to honor the promised service guarantee. In exchange, premium customers must pay a membership fee to maintain their premium status. We assume that customers purchase their memberships in a sequencing order, and our model provides a way of calculating the bottom-line price on the membership. For both the analysis and numerical results, we conclude managerial insights, from both the viewpoint of a customer and a company, on when to purchase the membership, and how much the membership fee will be.

We organize the remainder of the paper as follows. In Section 2, we present the notation and the chance-constrained CVRP model to solve CVRP under demand uncertainty when service guarantees are concerned. In Section 3, we then present the numerical results on nine commonly used CVRP instances, `A-n32-k5`, `A-n55-k9`, `A-n80-k10` from Augerat et al. (1998), `M-n200-k16` and `G-n250-k25` from Gillett and Johnson (1976) and `E-n76-k7`, `E-n76-k8`, `E-n76-k10` and `E-n76-k14` from Christiansen and Jens (2007), to represent the settings of small, medium, and large distribution networks. In Section 3.1, we demonstrate the strategic advantage of the chance-constrained CVRP model over the stochastic CVRP with recourse. The strategic advantage includes scheduling complexity, flexibility, and insights for customers. In Section 3.2, we present an alternative CVRP policy for handling the demand uncertainty and achieve a 100% delivery guarantee. The policy prioritizes delivery for premium customers at an early time, and such a policy must have a routing plan to start with. We compare the performances of the two approaches along with the policy. In Section 4, we analyze the pricing of premium membership as an incremental increase of expected costs. We present conclusions in Section 5.

2. The capacitated vehicle routing problem with chance constraints

We consider a central depot $i = 1$ and a set of $i = 2,\ldots,n$ customers. To satisfy the demands of $n-1$ customers, we have a fleet of m homogeneous vehicles, which means that the vehicles are of the same type with a capacity of U. The demand of a customer i is denoted by $d_i, i = 2,\ldots,n$. Suppose that these customers' locations are on a graph $G = (V,A)$, where V is the set of n nodes (i.e., the locations of the central depot and $n-1$ customers), and A is the set of arcs. Let $C = (c_{ij})$ be a cost matrix associated with A. Matrix C is symmetric when $c_{ij} = c_{ji}, i \neq j$ for any arc $(i,j) \in A$, and asymmetric otherwise. The travel cost is $c_{ij}, i \neq j$ between every pair of locations, and to avoid dummy shipping from and to the same location, we need to set $c_{ii} = \infty$, practically, a large number. The value of c_{ij} is proportional to the distance between customers i and j. We also assume $c_{ij} + c_{jk} \geqslant c_{ik}$ for any $i,j,k \in V$, i.e., C is said to satisfy the triangle inequality, and there is enough time in the day while customers have a wide time window for delivery.

The capacitated vehicle routing problem involves finding the routing and shipping decision with visits to customer locations from the central depot (the depot, hereafter) by all m vehicles. Without violating the vehicle capacity, vehicles will travel from the depot to customer locations and will return to the depot by the end of the planning period. The objective is to minimize the total shipping cost.

If the model is feasible, the shipping volume to the customer locations would completely fulfill the demands. To describe the routing decision, we need the following binary variables x_{ijk}:

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ visits customer } j \text{ immediately after node } i \\ 0 & \text{otherwise} \end{cases}$$

For the shipping volume, let y_{ijk}, a nonnegative real valued variable, represent the shipping volume from location $i = 1,...,n$ to customer $j = 2,...,n, j \neq i$ by vehicle $k = 1,...,m$. The capacitated vehicle routing model is:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\sum_{k=1}^{m} x_{ijk} \\
\text{subject to: } & x_{ijk} \in \{0,1\}, y_{ijk} \geqslant 0, \quad \forall (i,j) \in A, \ i = 1,...,n, \ j = 2,...,n, \ i \neq j, k = 1,...,m \\
& \sum_{i=1}^{n}\sum_{k=1}^{m} x_{ijk} = 1, \quad j = 2,...,n \quad \text{(one visit)} \\
& \sum_{i=1}^{n} x_{ipk} - \sum_{j=1}^{n} x_{pjk} = 0, \quad k = 1,...,m, \ p = 1,...,n \quad \text{(return)} \\
& \sum_{j=2}^{n} x_{1jk} \leqslant 1, \quad k = 1,...,m \\
& u_i - u_j + n\sum_{k=1}^{m} x_{ijk} \leqslant n-1, \quad \text{for } 2 \leqslant i \neq j \leqslant n \quad \text{(subtour elimination constraints)} \\
& \sum_{i=1, i\neq j}^{n}\sum_{k=1}^{m} y_{ijk} \geqslant d_j, \quad j = 2,...,n \quad \text{(demand satisfaction)} \\
& \sum_{i=1}^{n}\sum_{j=2, i\neq j}^{n} y_{ijk} \leqslant U, \quad k = 1,...,m \quad \text{(vehicle capacity)} \\
& y_{ijk} \leqslant M x_{ijk}, \quad j = 2,...,n \quad \text{(volume on route)}
\end{aligned}$$

where the Subtour Elimination Constraints (SEC) are designed to prevent the formation of subtours not including the depot. We use the SEC format from Svestka and Huckfeldt (1973) and Gavish (1976). The variable u_i is the real valued variable, which represents the load of the vehicle after visiting customer i. M is an artificially large number to ensure that the volume will only happen on the scheduled route. This is a mixed integer programming model when d_i is known.

Now, we study the capacitated vehicle routing problem under random demand $[d_2;...;d_i;...;d_n]$, which is the vector of $n-1$ jointly distributed random variables. We do not assume that the demands are component-wise independent, i.e., any pair of d_i, d_j, $i \neq j$ is independent. The decision-maker wants to maintain a certain service guarantee $1-\alpha \in [0,1]$, such as with a 95% of chance, as an assurance to fulfill customer demands, regardless of demand uncertainty. For example, the decision-makers commit to the 95% service guarantee to customers that are under 95% of chance, the fleet delivery will satisfy all of the demands, and no delays or shortages will be perceived by any customers. To model such an optimization problem with the service guarantee, we need a technique called chance-constrained optimization. We have

$$\begin{aligned}
\min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\sum_{k=1}^{m} x_{ijk} \\
\text{subject to: } & x_{ijk} \in \{0,1\}, y_{ijk} \geqslant 0, \ \forall (i,j) \in A, \ i = 1,...,n, \ j = 2,...,n, \ i \neq j, \ k = 1,...,m \\
& \sum_{i=1}^{n}\sum_{k=1}^{m} x_{ijk} = 1, \quad j = 2,...,n \quad \text{(one visit)} \\
& \sum_{i=1}^{n} x_{ipk} - \sum_{j=1}^{n} x_{pjk} = 0, \quad k = 1,...,m, \ p = 1,...,n \quad \text{(return)} \\
& \sum_{j=2}^{n} x_{1jk} \leqslant 1, \quad k = 1,...,m \\
& u_i - u_j + n\sum_{k=1}^{m} x_{ijk} \leqslant n-1, \quad \text{for } 2 \leqslant i \neq j \leqslant n \quad \text{(subtour elimination constraints)} \\
& \mathbb{P}\left(\sum_{i=1, i\neq j}^{n}\sum_{k=1}^{m} y_{ijk} \geqslant d_j, j = 2,...,n\right) \geqslant 1-\alpha \quad \text{(demand satisfaction)} \\
& \sum_{i=1}^{n}\sum_{j=2, i\neq j}^{n} y_{ijk} \leqslant U, \quad k = 1,...,m \quad \text{(vehicle capacity)} \\
& y_{ijk} \leqslant M x_{ijk}, \quad j = 2,...,n \quad \text{(volume on route)}
\end{aligned}$$

This problem is a mixed integer programming problem with nonlinear constraints. It is a combinatorial optimization model, and there is no tractable method for reaching an optimal solution. In reality, it is a difficult problem with respect to two aspects: the discontinuity of the feasible region, and the nonconvexity of the chance constraint. Thus, such a model is nearly impossible to solve in both theory and reality, in its current form.

We need to solve this problem by parts. There are three difficulties regarding our capacitated vehicle routing problem with a service guarantee. The first difficulty involves imposing the service guarantee on demand uncertainty by adopting the chance constraint. This chance constraint is usually nonlinear, nonconvex, and even discontinuous, which is problematic for numerical methods. Moreover, the demand distributional information is not available in reality. For example, a logistics company usually has a massive amount of data over time, and the distribution is rather empirical. The data would be unlikely to fit any known distribution. This difficulty invalidates most assumptions on the demand distribution, as well as the approximation method, by replacing the chance constraint with some deterministic counterparts. Even with the removal of the chance constraint, the remaining problem is still a mixed integer programming, and the exact solution technique will only be available for small or mid-scale problems.

We notice that within the chance constraint, decisions solely involve the shipping volume by customers, which are real-valued decision variables. We decide to find $z := [z_2;\ldots;z_i\ldots;z_n]$, such that whenever we have

$$\sum_{i=1,i\neq j}^{n}\sum_{k=1}^{m} y_{ijk} \geqslant z_j, \quad j = 2,\ldots,n,$$

the chance constraint

$$\mathbb{P}\left(\sum_{i=1,i\neq j}^{n}\sum_{k=1}^{m} y_{ijk} \geqslant d_j, j = 2,\ldots,n\right) \geqslant 1-\alpha$$

will be satisfied. Thus, we have the following model

$$\begin{aligned} &\min \quad \sum_{j=2}^{n} z_j \\ &\text{subject to: } \mathbb{P}(z_j \geqslant d_j, j = 2,\ldots,n) \geqslant 1-\alpha. \end{aligned} \tag{2.1}$$

The purpose of Model (2.1) is to replace the problematic chance constraint with a deterministic linear constraint by calculating z^*, which is the solution of Model (2.1). We adopt the solution technique in Chen et al. (2017), and the method will solve the problem reasonably quickly when the distribution of d is log-concave (without assuming component independence) because the chance constraint

$$\mathbb{P}(z_j \geqslant d_j, j = 2,\ldots,n) \geqslant 1-\alpha$$

may be less troublesome when the following condition is satisfied.

Lemma 1. *When the random vector of demands for $n-1$ customers, i.e., $[d_2;\ldots;d_i;\ldots;d_n]$, is log-concave, the chance constraint has a convex equivalent.*

This result is in Prékopa (1995) and many other papers, such as the log-concavity in Prékopa (1980). However, the distribution of the demand is rather empirical. To satisfy the above condition, the distribution needs to be continuous and log-concave. By the Clivenko-Cantelli Theorem (see Van der Vaart, 2000), the empirical distribution function estimates the cumulative distribution function and converges with probability 1. The empirical distribution can be presented as an underlying continuous distribution. Once the empirical distribution is in the format of a continuous distribution, we would assume log-concavity because so many commonly used distributions are indeed log-concave. For example, the normal distribution, uniform distribution, gamma distribution with a shape parameter greater than 1, beta distribution with all parameters greater than 1, Weibull distribution, Laplace distribution, logistic distribution, exponential distribution, and extreme value distribution are all log-concave. There are very few commonly used distributions that are not log-concave, such as the lognormal distribution, Chi-square distribution, and t-distribution, which are often used to describe the distributions of various statics rather than random variables arising from real problems. The author showed that the method would converge to the true optimal asymptotically. Since these details are rather technical, we refer readers to, references therein.

Once we have the optimal solution of Model (2.1), i.e., z^*, we remove the chance constraint and the random demand d from the original model. We have

$$\min \quad \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\sum_{k=1}^{m} x_{ijk}$$

$$\text{subject to: } x_{ijk} \in \{0,1\},\ y_{ijk} \geqslant 0\ ,\quad \forall\ (i,j) \in A,\ i = 1,\ldots,n,\ j = 2,\ldots,n,\ i \neq j,\ k = 1,\ldots,m$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m} x_{ijk} = 1,\quad j = 2,\ldots,n \quad \text{(one visit)}$$

$$\sum_{i=1}^{n} x_{ipk} - \sum_{j=1}^{n} x_{pjk} = 0,\quad k = 1,\ldots,m,\ p = 1,\ldots,n \quad \text{(return)}$$

$$\sum_{j=2}^{n} x_{1jk} \leqslant 1,\quad k = 1,\ldots,m$$

$$u_i - u_j + n\sum_{k=1}^{m} x_{ijk} \leqslant n-1,\quad \text{for } 2 \leqslant i \neq j \leqslant n \quad \text{(subtour elimination constraints)}$$

$$\sum_{i=1}^{n}\sum_{j=2,i\neq j}^{n} y_{ijk} \leqslant U,\quad k = 1,\ldots,m \quad \text{(vehicle capacity)}$$

$$y_{ijk} \leqslant Mx_{ijk},\quad j = 2,\ldots,n \quad \text{(volume on route)}$$

$$\sum_{i=1,i\neq j}^{n}\sum_{k=1}^{m} y_{ijk} = z_j^*,\quad j = 2,\ldots,n$$

This model is a mixed integer program, which can be solved with a solver, but would be computationally intractable when the problem is large. There are many heuristics for the model in obtaining a satisfactory solution of routing and shipping volume decisions for all $n-1$ customers. In this research, we adopt the tabu search metaheuristic.

3. Numerical results

In this section, we examine the computational requirements and solution quality of the proposed two-phase approach to solving chance-constrained CVRP problems. The test problems consist of capacitated vehicle routing problems in the literature, including the small and medium scale problems of Augerat et al. (1998), `A-n32-k5`, `A-n55-k9`, and `A-n80-k10` and `E-n76-k7`, `E-n76-k8`, `E-n76-k10` and `E-n76-k14` from Christiansen and Jens (2007). The n refers to the number of customers, and the k refers to the number of vehicles available. The large-scale test problems consist of `M-n200-k16` and `G-n250-k25` from Gillett and Johnson (1976). We made some changes to the selected problem instances, as well. Since the original examples have only deterministic customer demands, we replace these deterministic customer demands with a vector of normal random variables, whose components are jointly distributed with the deterministic demands as their mean values. The correlation matrix has diagonal components valued at 1, while the off-diagonal values are 0.5. We note that the joint distribution of the random vector would have a broad spectrum of choices as long as the distribution is log-concave, which is a requirement for preserving the convexity of the model. For instances `E-n76-k7`, `E-n76-k8`, `E-n76-k10` and `E-n76-k14`, the stochastic demands are Poisson with the nominal demand as the mean.

We choose these instances for two reasons. First, these problems with known demands are studied intensively with the best-known solutions. The advantage of adopting stochastic modeling would be rather simple and straightforward. Second, we choose these problem instances with a range in the number of customers, as well as the number of vehicles to represent small, medium, and large distribution networks. Through numerical experiments, we need to compare the performances of the chance-constrained CVRP, the chance-constrained CVRP with recourse, and CVRP with recourse under different service levels. We must show that the chance-constrained CVRP will gain strategic advantages to CVRP with recourse by reducing reloading trips with a limited increase in vehicles and inventory.

The solutions are achieved on two different computing platforms. The chance-constrained CVRP method was solved on an Intel i7 4770 K processor running Ubuntu 14.04. The tabu search metaheuristic was executed on an Intel i7 5930 K processor running at 4.4 GHz. The solution time of the chance-constrained CVRP method is less than 1 h on small and medium-scale problems, but increases exponentially for large-scale problems. The tabu search metaheuristic solves small and medium-scale problems in a fraction of a minute and large-scale problems in less than 2.3 min.

3.1. The comparison between the chance-constrained CVRP method and CVRP with recourse method

Two defined problem classes of low and high variance were created by assuming a standard deviation of 1 in the low variance case, and 25% of the mean in the high variance case. Demands of 5 or less were assigned a standard deviation of 1. For `E-n76` instances, customer demands are Poisson with a different number of vehicles. We thereby merely report the computational performance of serving all customers as premium customers. The test problems were further expanded by considering the number of premium customers for whom a service guarantee of 95% would be enforced. Table 1 shows the solutions obtained on the low variance test problems using the proposed two-phase approach. The first phase is the delivery amount allocated to each customer by solving the chance-constrained CVRP method. Phase 2 consists of the reactive tabu search metaheuristic of Chiang and Russell (1997), which determines the allocation of customers to routes and the visitation sequence on the routes. The tabu search

Table 1
Chance-constrained computational results on low variance test problems with a service guarantee of 95%.

Instance	Premium customers	Optimal distance	Vehicle required	Optimization CPU time	Tabu search CPU time
A-n32-k5	None	784	5	0	0.01
A-n32-k5	1, 2	787.81	5	0	0.01
A-n32-k5	1, 2, 3, 4	801.64	5	0.087	0.01
A-n32-k5	1,2,...,10	835.37	5	1.113	0.01
A-n32-k5	All	899.79	6	5.75	0.01
A-n55-k9	None	1073	9	0	0.02
A-n55-k9	1, 2	1075.5	9	0	0.02
A-n55-k9	1, 2, 3, 4	1075.17	9	0.088	0.02
A-n55-k9	1,2,...,10	1104.32	9	1.126	0.02
A-n55-k9	All	1245.69	11	11.35	0.03
E-n76-k7	All	811.79	10	44.28	0.06
E-n76-k8	All	911.09	12	46.25	0.05
E-n76-k10	All	1056.22	15	47.22	0.04
E-n76-k14	All	1334.80	21	46.89	0.04
A-n80-k10	None	1763	10	0	0.07
A-n80-k10	1, 2	1786.9	10	0	0.07
A-n80-k10	1, 2, 3, 4	1797.14	10	0.088	0.07
A-n80-k10	1,2,...,10	1822.94	10	1.105	0.07
A-n80-k10	All	2194.05	13	49.65	0.07
M-n200-k16	All	1535.73	20	557.04	1.26
G-n250-k25	All	5959.80	27	1491.46	2.28

metaheuristic was designed to solve CVRP with time windows. The key feature of the metaheuristic used in this study are (i) parallel construction of initial routes. The parallel construction heuristic is similar to the sequential insertion heuristic in Solomon (1987), but is applied to m emerging routes rather than one at a time; (ii) invoking the improvement process during construction; (iii) intensification strategies for the tabu search; (iv) diversification strategies; and (v) reactive tabu search.

The tabu search improves the current solution by executing moves within a specified neighborhood of the current solution. The proposed solution procedure uses pairwise node exchanges between locations on different routes. This node exchange procedure is attributed to Osman (1993) and is called the λ-interchange mechanism. The λ-interchange moves consist of three possible node exchanges: insert, delete, and swap. Not every movement is feasible. The feasibility for the movements of nodes is dictated by vehicle capacity.

Table 1 shows the number of vehicles required by the tabu search for the delivery loads determined by the chance-constrained CVRP method. The table also shows the total distance traveled and the computation time required by the chance constraint and tabu search solution methods. For problem instances in which the number of premium customers is 2, 4, or 10, the number of vehicles required is the same as that for the associated deterministic problem. However, when all customers are chance-constrained, the additional demand allocated to achieve a 95% overall customer service guarantee increases the number vehicles required. We note that a 95% service level for all customers would not be necessary in reality because, for a CVRP problem of 100 customers, under a service level of 95%, individual customers' service level would be close to 100%, even if customer demands are correlated. It also leads to a tremendous increase regarding the number of vehicles and, safety inventory. In the later section, we present our simulation study with a lower service level.

To satisfy the promised service guarantees, a common practice involves satisfying customers' uncertain demands through reloading trips when a shortage occurs. This method is referred to as the CVRP with recourse (i.e., two-stage) approach in the literature, such as Laporte et al. (1992), Dror et al. (1989) and references therein. The CVRP with recourse approach ensures that whenever a shortage occurs, the logistics company will deliver the goods at a later time by scheduling reloading trips. The previously set delivery schedules will be subject to changes whereby fleet vehicles will be sent back to the depot to reload the volume of the estimated shortage and deliver the goods by the end of the day as reloading trips. While the recourse approach represents a 100% service guarantee, the reloading trips would add scheduling complexity and cost inefficiency. The schedule has two stages. The first stage concerns the fleet schedule before uncertain demands are realized, and the second stage decision concerns the reloading trips. The objective function of the recourse model is the expected total cost, which represents the sum of the first-stage routing cost and the expected service recovery cost due to recourse. From the academician's viewpoint, recourse-based CVRP is a two-stage stochastic mixed-integer programming problem, which would be solved numerically by the methods in Sen (2005).

The chance-constrained CVRP and CVRP with recourse are different in many ways. First, the routing plan of the chance-constrained model is fixed over time, and nearly no reloading trips are necessary. For the recourse method, however, reloading trips are a secondary means of satisfying the customer demands. The recourse method exhibits large variations in the total travel distances and the number of reloading trips. Occasionally, drivers may be forced to work overtime or deliver on the second day due to drivers' working hour limitations or heavy traffic congestion on the first day. Second, the chance-constrained CVRP method carries a large volume to avoid the necessity of reloading trips. As a result, the chance-constrained CVRP may not be a good choice for the

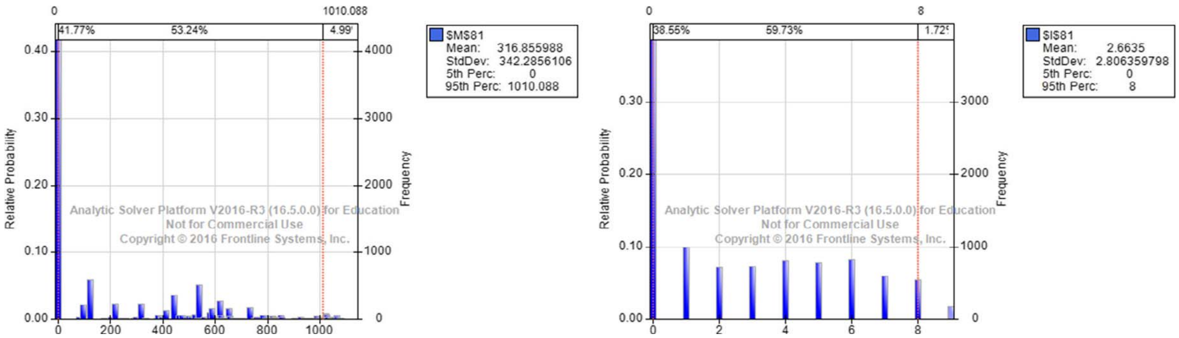

Fig. 1. Two-stage recourse results on `A-n80-k10`, low variance case.

distribution network of perishable goods, as there will be the variable cost associated with the shipping volume. For example, the distribution network of perishable fruit would adopt the recourse model because the vehicles may return less unsold fruit to the depot after an entire day spent on the road.

Our chance-constrained CVRP model has an advantage from the viewpoint of operations managers, in that the chance-constrained routing solution provides a fixed delivery plan, and a reloading trip for premium customers is highly unlikely. That is, the chance-constrained method brings stability to the CVRP problem so that the logistics company would prefer a reloading free delivery plan. Thus, the comparison between our method and its alternatives will focus on the number of reloading trips and overall costs. In Fig. 1, the left graph represents the additional travel distance, of the recourse method against the original deterministic route. The chance-constrained CVRP approach required a fixed 2194.05 distance whereas the recourse approach has a mean of $1769.72 + 320.16 = 2089.88$ and a max of 2901.39. The right graph represents the number of reloading trips required by the CVRP with recourse method for the problem instance `A-n80-k10`. The recourse approach has a much higher variance in terms of the number of vehicles and or depot revisits, compared to the fixed 13 required by the chance constrained approach. However, for 51.53% of the time, it required 12 or fewer vehicles, and for 35.83% of the time, it got by with the original 10 vehicles. In the worst case scenario, the CVRP with recourse approach required 10 vehicles and 9 visits back to the depot to fulfill a service guarantee for all customers. Thus, it seems that the number of reloading trips is a primary concern in using the CVRP with recourse method. Similar results occur in the small and medium networks, as well. We present the results in Figs. 2 and 3 with the same layout as in Fig. 1.

Tables 2 and 3 show the simulation results for the CVRP with recourse method. In the low variance case, this approach requires fewer vehicles, but at the expense of many more expected reload trips to the depot. The routes are highly variable, and in terms of the distance traveled, are outperformed by the chance-constrained CVRP approach. However, in the high demand variance case, the CVRP with recourse approach requires significantly less travel distance and fewer vehicles plus reload trips, except in the rarest and most extreme cases. In the high demand variance case, the two-stage with recourse approach outperforms the 95% chance-constrained CVRP.

The following set of experiments deals with the case of all customers requiring 100% delivery guarantees. Tables 4 and 5 show the impact of low and high customer demand variances for the chance-constrained CVRP method. We solve the chance-constrained CVRP for a shipping plan of a 95% service level for all customers. We do not expect many reloading trips under this plan. However, if there is a shortage, we will arrange reloading trips accordingly. We refer to this method as the chance-constrained CVRP with recourse. In the low variance case, when we place premium customers earlier in the route, the revised routing plan achieved a 100% delivery guarantee, as shown in the simulation study. We also note that there was never an instance of where recourse to reload at the depot was required because we increase the number of vehicles as the implementation of the chance-constrained CVRP solution. This results in relative efficiency and route stability. In the high variance case, very few reloading trips were required, but an increase in the number of vehicles was not competitive in the case of the CVRP with recourse method. Thus, the variance of customer demands may

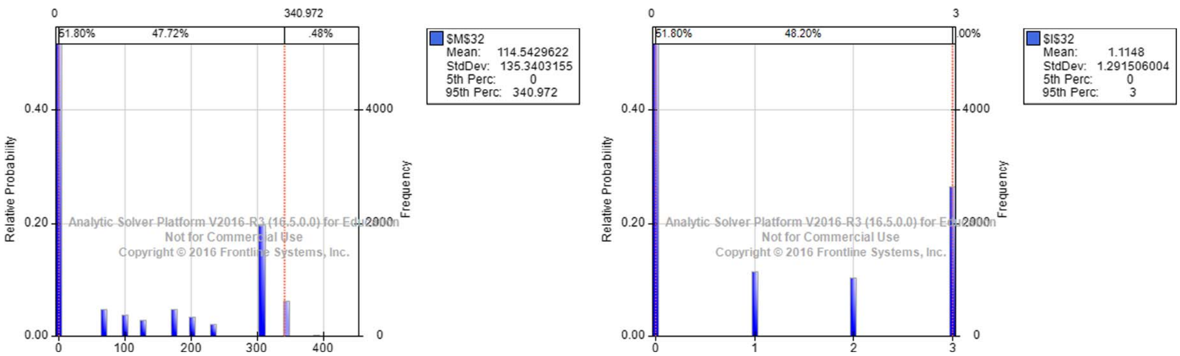

Fig. 2. Two-stage with recourse results on `A-n32-k5`, low variance case.

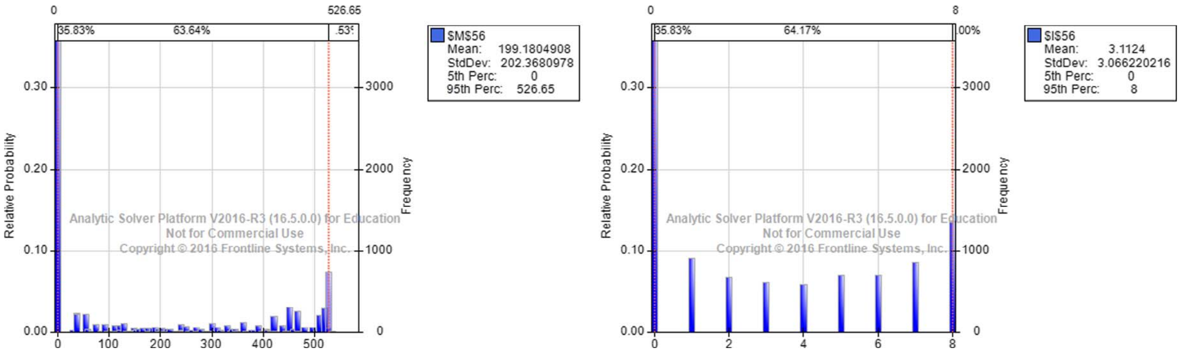

Fig. 3. Two-stage with recourse results on `A-n55-k9`, low variance case.

Table 2
Results of the CVRP with recourse, low variance cases.

Instance	Premium customer	Mean total distance	Vehicles	Mean reloading trips	Mean reloading distance
A-n32-k5	All	901.62	5	1.113	114.428
A-n55-k9	All	1273.64	9	3.112	199.180
A-n80-k10	All	2089.88	10	2.664	320.161
M-n200-k16	All	1588.97	16	7.399	228.159
G-n250-k25	All	6652.00	25	7.770	1050.978

Table 3
Results of the CVRP with recourse, high variance cases.

Instance	Premium customer	Mean total distance	Vehicles	Mean reloading trips	Mean reloading distance
A-n32-k5	All	949.45	5	1.332	162.377
A-n55-k9	All	1336.48	9	3.711	262.024
A-n80-k10	All	2235.57	10	3.826	465.849
M-n200-k16	All	1668.52	16	7.854	307.709
G-n250-k25	All	7390.14	25	11.134	1789.124

greatly impact the performance of the routing plan. When the variance is relatively moderate, the chance-constrained CVRP model will make a fixed routing plan that deploys fewer vehicles to achieve a high level of service guarantee without arranging for a reloading trip. When demand uncertainty is intense, the fixed routing plan may be changed by adding a significant number of vehicles to assure the same level of delivery guarantee and to avoid reloading trips.

3.2. An assessment of service guarantee approaches and policies

The on-time delivery or service guarantee of products and services is of strategic importance to companies as both a competitive weapon and a survival instrument. Customers feel more satisfied if products or services are delivered on time. The chance constrained approach ensures that the logistics company will satisfy all premium customer demands with a high probability, e.g., 95% over time. We assume that customer demands are highly jointly imposed and highly correlated by their nature. Also, such a guarantee is for all premium customers as a whole. Thus, statistically speaking, it implies that each customer may experience almost perfect customer satisfaction, except for some extremely rare occasions. This is the advantage of the chance-constrained approach for CVRP, and once a routing plan is ready to implement, the company's plan will be mostly fixed and subject to few changes.

When the demand is uncertain, in addition to our chance-constrained optimization method, many logistics companies use

Table 4
Chance-constrained derived 100% delivery guarantee for all customers, low variance cases.

Instance	Premium customer	Mean total distance	Vehicles	Mean reloading trips	Mean reloading distance
A-n32-k5	All	899.79	6	0	0
A-n55-k9	All	1245.69	11	0	0
A-n80-k10	All	2194.05	13	0	0
M-n200-k16	All	1535.73	20	0	0
G-n250-k25	All	5959.80	27	0	0

Table 5
Chance-constrained derived 100% delivery guarantee for all customers, high variance cases.

Instance	Premium customer	Mean total distance	Vehicles	Mean reloading trips	Mean reloading distance
A-n32-k5	All	1230.26	8	0.0002	0.0295
A-n55-k9	All	1720.69	17	0.0037	0.2599
A-n80-k10	All	3132.16	20	0	0
M-n200-k16	All	2456.98	41	0	0
G-n250-k25	All	9777.26	49	0	0

Table 6
Chance-constrained derived 100% delivery guarantee with limited premium customers.

Instance	Premium customer	Mean total distance	Vehicles	Mean reloading trips	Extra distance incurred
A-n32-k5	1, 2	787.81	5	0	0
A-n32-k5	1, 2, 3, 4	816.02	5	0	14.38
A-n32-k5	1,2,...,10	859.18	5	0	23.81
A-n55-k9	1, 2	1075.5	9	0	0
A-n55-k9	1, 2, 3, 4	1177.99	9	0	2.825
A-n55-k9	1,2,...,10	1182.50	9	0	78.18
A-n80-k10	1, 2	1864.55	10	0	77.65
A-n80-k10	1, 2, 3, 4	1872.74	10	0	75.60
A-n80-k10	1,2,...,10	1801.72	10	0	0

delivery time guarantees as their market positioning strategy. Urban (2009) reviews delivery time guarantees and provides an analytical approach for establishing optimal delivery-time guarantee and pricing policies for a firm making uniform guarantees on the delivery and provision of their services. Many companies adopt this idea and establish premium customer service guarantees for customers who are willing to pay extra fees. One example is Amazon's Prime membership, which includes two-day shipping for most items, or even same-day delivery for selected items. Chatterjee et al. (2002) consider the interdependence between marketing and operations functions and note that logistics companies may find themselves committing early to a delivery commitment, only to find later that operational constraints prevent them from satisfying such guarantees. Thus, it is critical to have a delivery system to execute the promised service guarantees.

In this section, we continue to conduct a simulation study of the effectiveness and resource costs of 95% chance constrained service and 100% delivery guarantees, i.e., the chance-constrained CVRP with recourse. We aim to highlight the advantage of the chance-constrained CVRP method because we realize that demand shortages will be eliminated, at least according to the simulation study, by implementing the routing solution of the chance-constrained CVRP method to achieve a 100% delivery guarantee. In problem instances in which there are relatively few premium customers, 100% delivery guarantees can be achieved by a route modification that places premium customers earlier in the routing plan obtained by the chance-constrained CVRP method.[1] To test the effectiveness of the service guarantee approaches under different levels of demand uncertainty and network scales, simulation experiments were conducted in which the route structure and stochastic demand levels were simulated in an Excel spreadsheet. The simulation software used was the Analytic Solver Platform from Frontline Systems. The demand was correlated as discussed.

Table 6 shows the results of using the 95% service guarantee as a basis of achieving 100% delivery guarantees for problems with a limited number of premium customers. No recourse in terms of reloading trips to the depot is required when the number of premium customers is small relative to the total number of customers. Simulation experiments as well as intuition indicate that when route delivery failures occur, they are at or near the end of the route. The chance-constrained CVRP of a 95% service guarantee may achieve a 100% delivery guarantee by modifying the obtained solution on the customer sequence by placing premium customers before the end of the route. The simulation results in Table 6 indicate that little route modification was required to achieve zero delivery failures. Although there are no analytical results to guarantee 100% delivery levels for these test problems, there were no delivery failures in 10,000 simulation days, which is equivalent to more than 27 years of operation. A more systematic approach would be to employ a vehicle routing solver that handles time windows. Premium customers would be assigned a wide time window, but one that precludes delivery very late in the day. Non-premium customers would be assigned the widest time window possible. Thus, in the case of a limited number of premium customers, reloading trips can be eliminated. By implementing the solution from the chance-constrained CVRP, the logistics company may effectively eliminate reloading trips and stick to a fixed schedule when the service guarantee is only promised to a small group of premium customers.

In the previous subsection,Tables 4 and 5 show that there are extremely few required recourse trips to the depot in 10,000 simulation

[1] Extra route distance required to achieve a 100% service guarantee relative to the chance-constrained original 95% routes.

trials, even when the variation is high. However, significantly more vehicles and travel distance are required. This is particularly true in the case of large-scale problems, e.g., $n \geqslant 100$. Imposing the 95% overall service guarantee is too restrictive when servicing a large number of customers. For example, if the demand is independent, then a 95% overall service guarantee for all customers imputes a 99.98% service guarantee for an individual customer. This is somewhat extreme and counter to current trends toward lean production and low inventory levels. For large-scale problems, vehicles would return to the depot with a combined load equal to roughly 100% of total demand. This would not be practical with a perishable product and leads to an insight evidenced by the simulation study.

We now consider the routing decision under a much lower service level for all customers, given that the goal of vehicle routing for most logistics companies is to pursue a fixed schedule with fewer vehicles and less safety inventory required. We also realize that a significantly lower system service level will imply a moderate, at most, reduction in each individual customer's service level. That is, we still expect a very limited number of reloading trips from the routing decision of the chance-constrained CVRP, even if the system service level is much lower. In Tables 7 and 8, we solve the chance-constrained CVRP at the 65% system service level for small problems, i.e., with fewer than 100 customers, and the 40% system service level for large problems with more than 100 customers.

The numerical results suggest that even if we use the routing plan of a chance-constrained CVRP with a system service level as low as 40% for both low and high variation cases, the mean reloading trips and the mean reloading distance are very limited. The logistics companies would moderately increase the number of vehicles and the amount of safety inventory in exchange for a largely fixed schedule. Thus, the chance-constrained CVRP would bring a strategic advantage to companies, regardless of the level of demand variation.

4. Premium customer membership

We showed in the numerical results that the service guarantee would improve the routing plan and gain a strategic advantage. However, such an advantage comes with a cost. When a logistics company promises a service guarantee to its premium customers, the total cost will surely increase because of the increased volume and the number of vehicles. In the chance-constrained CVRP model,when the problem grows larger, imposing a service guarantee for all customers will be increasingly unlikely because the value of z^* will grow very quickly. The projected overall cost would typically be too fast to sustain for logistics companies. From the numerical results, the increases in the total travel distance are more than 30%, and if we imposed service guarantee for all customers, every customer would be expected to pay 30% more. It is difficult to convince all customers to pay for such an increase because many customers need to control their business operations costs. For some customers, the service guarantee would not be able to justify a price increase by 30%. That is, not all customers are service level sensitive. Thus, there will be only a subset of customers willing to purchase the premium customer membership. Moreover, most customers will deliberate for a period of time before deciding whether or not to join the premium customer club.

In this section, we consider the problem in which the logistics company will only assure a service guarantee to a proper subset of all customers. A customer in such a subset is a premium customer. Since the service guarantee implies an increase in the total cost, the logistics company would charge the premium customer a membership fee in return. The value of a fair membership fee would be determined by the chance-constrained CVRP problem. Consider a set of customers $\mathscr{P} \subset \{2,\ldots,n\}$ such that a customer $i \in \mathscr{P}$ indicates that this customer would like to claim premium customer status. For all premium customers, the logistics company will impose a service guarantee on premium customers only. We have the following model:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\sum_{k=1}^{m} x_{ijk} \\
\text{subject to:}\quad & x_{ijk}\in\{0,1\},\ y_{ijk}\geqslant 0\,,\quad \forall\,(i,j)\in A,\ i=1,\ldots,n,\ j=2,\ldots,n,\ i\neq j,\ k=1,\ldots,m \\
& \sum_{i=1}^{n}\sum_{k=1}^{m} x_{ijk}=1,\quad j=2,\ldots,n \quad \text{(one visit)} \\
& \sum_{i=1}^{n} x_{ipk}-\sum_{j=1}^{n} x_{pjk}=0,\quad k=1,\ldots,m,\ p=1,\ldots,n \quad \text{(return)} \\
& \sum_{j=2}^{n} x_{1jk}\leqslant 1,\quad k=1,\ldots,m \\
& u_i-u_j+n\sum_{k=1}^{m} x_{ijk}\leqslant n-1,\quad \text{for } 2\leqslant i\neq j\leqslant n \quad \text{(subtour elimination constraints)} \\
& \mathbb{P}\left(\sum_{i=1,i\neq j}^{n}\sum_{k=1}^{m} y_{ijk}\geqslant d_j,\ j\in\mathscr{P}\subset\{2,\ldots,n\}\right)\geqslant 1-\alpha \quad \text{demand satisfaction} \\
& \sum_{i=1}^{n}\sum_{j=2,i\neq j}^{n} y_{ijk}\leqslant U,\quad k=1,\ldots,m \quad \text{(vehicle capacity)} \\
& y_{ijk}\leqslant Mx_{ijk},\quad j=2,\ldots,n \quad \text{(volume on route)}
\end{aligned}
$$

Similarly, we solve the following model to calculate $z^*_{\mathscr{P}} := \{z^*_j\}_{j\in\mathscr{P}}$.

Table 7
Chance-constrained CVRP with 40/65% system service levels with recourse for all customers, low variance cases.

Instance	Mean total distance	Vehicles	Mean reloading trips	Max reloading trips	Max reloading distance
A-n32-k5-65%	891.24	5	0.0022	3	1043.71
A-n55-k9-65%	1209.95	10	0.001	3	1370.41
A-n80-k10-65%	2216.72	12	0.9062	1	2227.73
M-n200-k16-40%	1454.30	19	0.0025	10	1811.49
G-n250-k25-40%	5848.62	26	0	0	0

Table 8
Chance-constrained CVRP with 40/65% system service levels with recourse for all customers, high variance cases.

Instance	Mean total distance	Vehicles	Mean reloading trips	Max reloading trips	Max reloading distance
A-n32-k5-65%	1092.49	6	1.255	5	1092.49
A-n55-k9-65%	1392.91	12	0.2273	9	1581.85
A-n80-k10-65%	2435.58	15	0.2254	13	4109.72
M-n200-k16-40%	1695.70	24	0.1603	24	2800.98
G-n250-k25-40%	7455.46	35	0	0	0

$$\begin{aligned} &\min && \sum_{j\in\mathscr{P}}^{n} z_j \\ &\text{subject to:} && \mathbb{P}(z_j \geqslant d_j, j\in\mathscr{P}\subset\{2,\dots,n\}) \geqslant 1-\alpha. \end{aligned} \tag{4.1}$$

and we solve the following model for the routing and shipping volume decisions.

$$\begin{aligned} &\min && \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\sum_{k=1}^{m} x_{ijk} \\ &\text{subject to:} && x_{ijk}\in\{0,1\},\ y_{ijk}\geqslant 0,\quad \forall\,(i,j)\in A,\ i=1,\dots,n,\ j=2,\dots,n,\ i\neq j,\ k=1,\dots,m \\ &&& \sum_{i=1}^{n}\sum_{k=1}^{m} x_{ijk}=1,\quad j=2,\dots,n \\ &&& \sum_{i=1}^{n} x_{ipk}-\sum_{j=1}^{n} x_{pjk}=0,\quad k=1,\dots,m,\ p=1,\dots,n \\ &&& \sum_{j=2}^{n} x_{1jk}\leqslant 1,\quad k=1,\dots,m \\ &&& u_i-u_j+n\sum_{k=1}^{m} x_{ijk}\leqslant n-1,\quad \text{for } 2\leqslant i\neq j\leqslant n \\ &&& \sum_{i=1}^{n}\sum_{j=2,i\neq j}^{n} y_{ijk}\leqslant U,\quad k=1,\dots,m \\ &&& y_{ijk}\leqslant Mx_{ijk},\quad j=2,\dots,n \\ &&& \sum_{i=1,i\neq j}^{n}\sum_{k=1}^{m} y_{ijk}=z_j^*,\quad j\in\mathscr{P} \end{aligned} \tag{4.2}$$

Since we only impose the service guarantee for a handful of customers, we expect that the increase of the overall cost would be limited. More importantly, if the logistics company collects membership fees from the premium customers, the membership fee revenue should, at least, break even with the cost increase. That is, customers would pay a membership fee to become a premium customer, who expects a service guarantee, regardless of the demand uncertainty. The logistics company, meanwhile, will increase revenue by offering service to premium customers.

We present the results to show the managerial implication that it will be less expensive to join the club of premium customers sooner than later. Suppose that the customers will become premium customers sequentially, i.e., one after another. To sustain a service guarantee of $1-\alpha$, we need to calculate the shipping volumes of premium customers by Model (2.1). For non-premium customers, their demands are assumed to be deterministic. Let $\mathscr{P}$ denote the set of premium customers. When there is only one premium customer i, i.e., $\mathscr{P}=\{i\}$, another customer j, $j\neq i$ wants to join the club. We solve the following models.

$$\min\{z_i \mid \mathbb{P}(z_i\geqslant d_i)\geqslant 1-\alpha\} \tag{4.3}$$

and

$$\min\{z_i + z_j | \mathbb{P}(z_i \geqslant d_i, z_j \geqslant d_j, i \neq j) \geqslant 1-\alpha\}. \tag{4.4}$$

Proposition 1. *Suppose the optimal solution of (4.3) is z_i^* and the optimal solution of (4.4) is $[\hat{z}_i^*; \hat{z}_j^*]$. We have $z_i^* \geqslant d_i, \hat{z}_j^* \geqslant d_j$, and $z_i^* \leqslant \hat{z}_i^*$ when* $1-\alpha > 0.5$.

Proof. z_i^* is a feasible solution of the model (4.3). Thus, we have $z_i^* \geqslant d_i$ when $1-\alpha > 0.5$ and $\mathbb{P}(z_i^* \geqslant d_i) = 1-\alpha$. By the argument of conditional probability, we have

$$\mathbb{P}(z_j \geqslant d_j) = \frac{\mathbb{P}(z_i \geqslant d_i, z_j \geqslant d_j, i \neq j)}{\mathbb{P}(z_i \geqslant d_i | z_j \geqslant d_j)} \geqslant 1-\alpha \tag{4.5}$$

and thus, we have $\hat{z}_j^* \geqslant d_j$ where $\mathbb{P}(\hat{z}_i^* \geqslant d_i, \hat{z}_j^* \geqslant d_j, i \neq j) = 1-\alpha$. Similarly, we have

$$\mathbb{P}(\hat{z}_i^* \geqslant d_i) = \frac{\mathbb{P}(\hat{z}_i^* \geqslant d_i, \hat{z}_j^* \geqslant d_j, i \neq j)}{\mathbb{P}(\hat{z}_j^* \geqslant d_j | \hat{z}_i^* \geqslant d_i)} \geqslant 1-\alpha = \mathbb{P}(z_i^* \geqslant d_i) \tag{4.6}$$

which implies $z_i^* \leqslant \hat{z}_i^*$. □

Proposition 2. *When there are multiple existing premium customers, adding another customer $j \notin \mathscr{P}$ will increase the shipping volumes for all premium customers.*

Proof. We show this result by induction from Proposition 1. Suppose the set of existing premium customers is $\mathscr{P}$ and $j \notin \mathscr{P}$. The existing shipping volume is $z^* \coloneqq [z_i^*;], i \in \mathscr{P}$ such that $\mathbb{P}(z_i^* \geqslant d_i, i \in \mathscr{P}) = 1-\alpha$, which minimizes the value of $\sum_{i \in \mathscr{P}} z_i^*$. When customer j joins the premium customers' membership club, the following model determines the shipping volume.

$$\left\{\min \sum_{i \in \widehat{\mathscr{P}}} z_i \,|\mathbb{P}(z_j \geqslant d_j, z_i \geqslant d_i, i \in \mathscr{P}) \geqslant 1-\alpha\right\} \tag{4.7}$$

where $\widehat{\mathscr{P}} \coloneqq \mathscr{P} \cup \{j\}$. Let $\hat{z}^* \coloneqq [z_i^*; z_j^*], i \in \mathscr{P}$ be the optimal solution of (4.7). By the argument of conditional probability, we have

$$\mathbb{P}(\hat{z}_i^* \geqslant d_i, i \in \mathscr{P}) = \frac{\mathbb{P}(\hat{z}_j^* \geqslant d_j, \hat{z}_i^* \geqslant d_i, i \in \mathscr{P})}{\mathbb{P}(\hat{z}_j^* \geqslant d_j | \hat{z}_i^* \geqslant d_i, i \in \mathscr{P})} \geqslant 1-\alpha = \mathbb{P}(z_i^* \geqslant d_i, i \in \mathscr{P}).$$

$[\hat{z}_i^*;]$, $i \in \mathscr{P}$ will be a feasible solution to (4.7) and by the definition of z^*, we have

$$\sum_{i \in \mathscr{P}} \hat{z}_i^* \geqslant \sum_{i \in \mathscr{P}} z_i^* \tag{4.8}$$

whenever $1-\alpha > 0.5$ for any number of premium customers. Thus, we have $[\hat{z}_i^*;] \geqslant [z_i^*;]$, $i \in \mathscr{P}$. □

The propositions jointly form an induction, which suggests that when a new customer joins the membership club, the shipping volumes of premium customers will be monotonically increasing with the shipping volumes of non-premium customers, unchanged. When a non-premium customer wants to join the club, the shipping volumes will increase not only for the new premium customers, but also for all existing premium customers. The overall cost will increase as a result, and it is fair to say that the cost increase is solely due to the change in the premium customer club. To cancel such a cost increase, the logistics company needs to charge a one-time membership fee to the incoming premium customer. In this research paper, we would help the decision-makers from both sides, the logistics company and the customer, to answer the following two questions. For the logistics company, the decision-makers need to determine a proper charge for the incoming premium customer. For the customer, when a logistics company announces a premium customer program, should the customer join the company immediately, or wait until a later time?

Suppose the set of premium customers is $\mathscr{P}$, and the new premium customer is j such that $j \notin \mathscr{P}$. Without customer j as a premium customer, the shipping volumes for premium customers are $[z_i^*; d_j;]$, $i \in \mathscr{P}, j \notin \mathscr{P}$ by solving Model (4.1). We solve Model (4.2) and the objective value will be the total cost. When customer j becomes a premium customer, the set of premium customers becomes $\widehat{\mathscr{P}} \coloneqq \mathscr{P} \cup \{j\}$. We solve Model (4.2) with the updated shipping volume, $[\hat{z}_i^*; d_j;]$, $i \in \widehat{\mathscr{P}}, j \notin \widehat{\mathscr{P}}$. By Proposition 1, we have $[z_i^*; d_j; i \in \mathscr{P}, j \notin \mathscr{P}] \leqslant [\hat{z}_i^*; d_j; i \in \widehat{\mathscr{P}}, j \notin \widehat{\mathscr{P}}]$, which means that the shipping volumes will increase for all premium customers. Let $\nu([z_i^*; d_j;], i \in \mathscr{P}, j \notin \mathscr{P})$ denote the objective value when customer j is not a premium customer, and $\nu([\hat{z}_i^*; d_j;], i \in \widehat{\mathscr{P}}, j \notin \widehat{\mathscr{P}})$ is the total cost when customer j joins the premium customers' club. The membership fee for customer j during this vehicle routing period is

$$\nu([\hat{z}_i^*; d_j;], i \in \widehat{\mathscr{P}}, j \notin \widehat{\mathscr{P}}) - \nu([z_i^*; d_j;], i \in \mathscr{P}, j \notin \mathscr{P}) \geqslant 0 \tag{4.9}$$

The membership should cover a period of time, e.g., yearly or monthly membership. If the vehicle routing period is each day, and there are 250 working days in each year, the membership price would bethe value of (4.9) factored 250 times for customer j. The calculation of (4.9) is mostly computational, and the decision-makers of the logistics company would have a bottom-line price quote for potential new premium customers.

It is rather difficult to extract the analytic results of (4.9) because Model (4.2) is a mixed integer program. The objective function is not even a continuous function. To gain managerial implications for customers, we need to relax the integer variables x_{ijk} in (4.2) by their continuous counterparts and reduce the problem complexity from a mixed integer program to a linear program. Let $\hat{\nu}([z_i^*;d_j;], i \in \mathscr{P}, j \notin \mathscr{P})$ be the optimal value of the relaxed Model (4.2) without customer j as premium customer and $\hat{\nu}([\hat{z}_i^*;d_j;], i \in \hat{\mathscr{P}}, j \notin \hat{\mathscr{P}})$ be the optimal value of the relaxed (4.2) with customer j in the premium customers' club. We have

$$\hat{\nu}([\hat{z}_i^*;d_j;], i \in \hat{\mathscr{P}}, j \notin \hat{\mathscr{P}}) - \hat{\nu}([z_i^*;d_j;], i \in \mathscr{P} j \notin \mathscr{P}) \geqslant 0 \tag{4.10}$$

as the approximated membership price.

We remark that the approximation by relaxation from (4.9) to (4.10) would not cause serious computational or theoretical challenges in calculating membership prices, although such a relaxation may be problematic in the literature. The reason is that our method adopts the chance-constrained optimization in determining the level of shipping volume under a particular service level, and during the solution process, the shipping schedule variables are not involved. Thus, the relaxation would not cause an issue at this step. We then use the metaheuristic to solve the vehicle routing problem in deciding on the shipping schedule. The metaheuristic in this paper is very mature, such that it delivers effective results.

Proposition 3. *$\hat{\nu}(\cdot)$ will be a non-decreasing, differentiable, and convex function with respect to shipping volumes.*

The proof would be shown through the duality results of the linear programming, and we choose to eliminate. The gradient of $\hat{\nu}(\cdot)$ would be a non-negative, and non-decreasing function. This result implies that the incremental increased rate of the membership fee will be non-decreasing, i.e.,

$$\nabla\hat{\nu}(\cdot) \geqslant \mathbf{0} \tag{4.11}$$

where **0** is a vector of corresponding dimensions. Besides the incremental increased rate of the membership fee, the increase in the shipping volumes,

$$[\hat{z}_i^*;d_j;i \in \hat{\mathscr{P}} j \notin \hat{\mathscr{P}}] - [z_i^*;d_j;i \in \mathscr{P} j \notin \mathscr{P}] \tag{4.12}$$

would be another factor affecting the membership fee because, by the Taylor theorem, (4.10) is approximated by

$$\nabla\hat{\nu}(\cdot)([\hat{z}_i^*;d_j;i \in \hat{\mathscr{P}} j \notin \hat{\mathscr{P}}] - [z_i^*;d_j;i \in \mathscr{P} j \notin \mathscr{P}]) \geqslant 0 \tag{4.13}$$

We thereby use the first-order Taylor extension to approximate the membership fee of customer j.

A premium customer usually requires a service guarantee. The demand uncertainties of different customers may vary significantly. Based on (4.13), we recommend that a management team should purchase premium membership as early as possible, in particular, for customers with strong demand variation. The demand uncertainty is the key factor for our recommendation. Consider an extreme case in which a customer has zero demand uncertainty. The overall cost increase by upgrading this customer to a premium customer would be zero. For a customer with strong demand uncertainty, however, the increase in the shipping volume, e.g., (4.12), would be significant, and so would the value of (4.13). If the management team purchases premium membership with many existing premium customers already, the incremental increase rate will be much greater in comparison to its counterpart during the early stage of the premium program.

We now present the numerical study regarding the premium membership fee. When the logistics company only assures customer demand satisfaction for premium customers rather than all customers, the total costs of the chance-constrained CVRP drop significantly. For instance, when imposing a service guarantee for 2, 4, and 10 out of 31 customers under a small network setting, the cost increases 0.49%, 2.25%, and 6.55% in comparison to the deterministic model, respectively. Similarly, for medium and large networks, increases in the chance-constrained CVRP when 2, 4, and 10 customers are premium customers, are up to 4%. This result provides insights for both the logistics company and customers. In Table 9, we find that the best strategy for joining a premium customer club is to take action early as a customer. For example, in a small or medium network, when the logistic company has no more than 2 premium customers, a premium membership would not cost much because the vast majority of customers are still operating under the assumed mean demand. The results also suggest that joining a premium customer club later might not be a bad idea either because the shipping volume or the cost of vehicle is partially covered by other customers. We can consider a negative membership fee, for the instance `A-n55-k9` with 4 premium customers as the error of approximating the membership fee by (4.13), in which the integer constraints are relaxed.

5. Conclusions and perspectives

In this paper, we examine a capacitated vehicle routing problem (CVRP) with stochastic demand. We solve the problem from the perspective of providing service guarantees to customers. We investigate and develop a methodology for premium membership

Table 9
The impact of a cost increase by assuring premium customers with a service guarantee of 95%.

Instance	Deterministic	2 PC	Membership	4 PC	Membership	10 PC	Membership	All	Membership
A-n32-k5	784	787.81	1.905	801.64	8.82	835.37	5.62	899.79	3.06
A-n55-k9	1073	1075.5	1.25	1075.17	−0.165	1104.32	4.86	1245.69	3.21
A-n80-k10	1763	1786.9	11.95	1797.14	5.12	1822.94	4.3	2194.05	5.37

pricing, and we provide insights regarding how it relates to delivery guarantees. We further provide suggestions on when the best time is for a customer to buy a premium membership.

The capacitated vehicle routing problem with stochastic demand is mathematically formulated as a chance-constrained problem to handle the levels of service guarantees required for customers. Two basic approaches are evaluated: the chance-constrained approach and the CVRP with recourse approach. A simulation model is developed to test the two approaches in terms of the resources required, efficiency, and satisfaction of delivery guarantees. In terms of service guarantees, we find that with a small number of premium customers, the routes designed by the chance-constrained CVRP approach can achieve 100% delivery guarantees by simple route modification and no recourse for reloading trips to the depot. However, when all customers are premium and require 100% delivery guarantees, the chance-constrained CVRP approach can require significantly more vehicles and delivery inventory than CVRP with recourse, especially for large-scale (more than 100 customers per period) problems in which customer demand exhibits high variance.

Our simulation study demonstrates that the chance-constrained CVRP approach has some strategic advantages over CVRP with recourse. The chance-constrained CVRP method would better serve a logistics company when the fixed routing plan and service guarantees are of concern. We present computational results on commonly studied small to large-scale CVRP instances. The chance-constrained CVRP approach and the recourse-based approach are different in many ways. First, the routing plan of the chance-constrained model is fixed over the time simulated, and nearly no reloading trips are necessary. For the recourse method, however, reloading trips are the primary means of satisfying customer demands. The recourse method exhibits large variations in the total travel distances and the number of reloading trips. Occasionally, drivers may be forced to work overtime or deliver on the second day due to drivers' working hour limitations or heavy traffic congestion on the first day. Second, the simulation experiments showed that the chance-constrained CVRP method generates superior solutions on the test problems when the variance in customer demand is relatively low. However, large variation instances require the change-constrained CVRP method in using significantly more vehicles and delivery inventory than CVRP with recourse. Given the relatively large amounts of delivery inventory prescribed by the chance-constrained CVRP method, it probably would not be a viable choice for the distribution of perishable goods.

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